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PHYSICAL MEASUREMENTS

DUFF and EWELL

BLAKISTON'S SCIENCE SERIES

PHYSICAL MEASUREMENTS

BY

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PREFACE.

Our intention in writing this book was not to give an account of physical laboratory methods in general, but to describe a limited number of carefully chosen exercises such as we have found in our experience to be suitable for the laboratory work of students who have had a fair course in General College Physics.

The descriptions of the exercises will usually fit apparatus and conditions of considerable diversity, but many practical details have been included where experience has shown that they are necessary. Other instructors who may adopt the book will probably find some of the exercises unsuited to their classes, but the list is sufficiently extensive to afford a considerable variety of selection.

The descriptions of apparatus are intended to be read by the student with the apparatus before him. Hence elaborate illustrations have been thought unnecessary. For an extended account of certain special topics, such as the theory of the balance and the construction of galvanometers, references to other works have been given.

Usually several text-books and special treatises have been referred to at the beginning of the account of an experiment. It is assumed that each student will have one of the text-books and that some of the special works will be found in the reference room of the laboratory. While the reference is generally to the latest edition (at the present date, 1910), those who have different editions will have no difficulty in finding the passages referred to. Each instructor who uses

the book will exercise his discretion as to what preliminary reading will be required and will issue the necessary instructions to his class.

We are indebted to Dr. Albert W. Hull for assistance in reading the page proof. Many of the tables have been taken from Ewell's Physical Chemistry.

CONTENTS.

PHYSICAL MEASUREMENTS.

INTRODUCTION.

1. Purpose of Course.

Intelligent work requires a clear perception of the end in view. It is important, therefore, to remember that the purpose of a course in Laboratory Physics is not only the attainment, by personal experimentation, of a more definite knowledge of the facts and principles of physics and an acquaintance with the use of measuring instruments and methods, but also the acquisition of a scientific habit of accuracy and carefulness in observing and examining phenomena and drawing conclusions therefrom.

2. General Directions.

Much time in the laboratory will be wasted unless some preparation be made before coming to the laboratory. The purpose and general method of the measurement to be made should be examined with the aid of the text-book and some of the references preceding the directions. This may usually be done in a few minutes at home, whereas it might require an hour or more in a laboratory where a number of people are moving around.

The readings made in the laboratory should always be recorded in a firmly bound book reserved for this purpose only, and never on loose slips of paper or in a book that may become dog-eared and untidy. When, for convenience or of necessity, two work together at an experiment, each should keep his own notes of the measurements made, and,

whenever possible, each should make a separate set of readings for himself, and these should be as independent as possible.

No operation should be performed or measurement made unless the purpose and meaning of it are understood; otherwise it may be made imperfectly or some essential part of it may be overlooked.

3. Reports.

An essential part of the work is a written report on each experiment completed. This should be handed in within a week after the work is finished. In preparing the report the writer has to make clear to himself the purpose and bearing of each part of the work and examine critically the value and accuracy of the final result. This exercise is as valuable as the experimental work itself. The report should be as brief as possible, consistently with giving the following information:

The purpose of the experiment (including the definition of the leading terms, such as coefficient of friction, mechanical equivalent of heat, etc.);

A brief statement of the method used;

A statement (tabulated if possible) of the observations and readings made:

An outline of the calculation of the final result (omitting the details of the numerical work);

A criticism of the reliability of the result;

Brief answers to the questions appended to the directions.

4. Errors.

A perfectly accurate experimental result is impossible; but some estimate can usually be formed as to the magnitude of the possible error and this is frequently of the greatest value. An experimental result of unknown reliability is often of very little value. Hence an estimate of

the accuracy of a measurement is very desirable in an account of the work.

Inaccuracy may arise from several different causes—(1) *errors of observation*, due to the inherent limitations of the observer's powers of observing and judging; (2) *instrumental errors*, arising from imperfections in the work of the instrument maker in constructing and subdividing the scale used by the observer; (3) *mistakes*, such as the mistaking of an 8 for a 3 on a scale; (4) *systematic errors* due to faultiness in the general method employed.

Instrumental errors may be decreased by using more accurate instruments or by *calibrating* the scales of the instruments used, that is, ascertaining and allowing for the errors in their graduation. This is frequently a difficult operation and unsuited for an elementary course. We shall, therefore, usually assume that the accuracy of the instruments is such that the instrumental errors are less than the errors of observation.

Mistakes in reading can be eliminated by care and repetition. Systematic errors are apt to arise when some indirect method of arriving at a result is adopted, a direct method being difficult or impossible. For example, the length of a wave of light cannot be measured directly and a method depending on diffraction or interference is usually employed (Exp. XXXIX). A careful study of the method used will often enable us to eliminate such errors by improving the details of the method, or, where this cannot be done, some estimate of the uneliminated errors can often be formed.

5. Errors of Observation.

Different methods of estimating the magnitude of errors of observation may be employed, the choice depending on the nature of the measurements. In many cases the quantity can be measured several times and the mean taken, it being probably more accurate than a single observation. In other cases circumstances do not permit repetition and a

single observation must suffice. In either case the observer can, from the circumstances of the case, say with a high degree of probability that the error cannot be greater than a certain magnitude. This we shall call the "possible error" of the measurement. It does not strictly mean the greatest possible error, since a greater error might be theoretically possible but very improbable.

(a) *When Only a Single Observation is Made.*—For example, a liquid, the temperature of which is varying slowly, is kept well stirred and the temperature is observed by means of a thermometer graduated to degrees. The temperatur at a certain time is noted as being between 36° and 37° and the observer, estimating to 0.1 of a division, records the temperature as 36.3°; but he does not trust his estimate closer than 0.1; that is, he considers that the real temperature may be as high as 36.4° or as low as 36.2°. He therefore states the temperature as 36.3° with a possible error of 0.1°, or $36.3° \pm 0.1°$. The actual error may, of course, be less than 0.1°; the latter is only a reasonable estimate of the limit of error of observation.

(b) *When Several Different Observations of a Quantity are Made.*—The mean of a number of observations of a quantity is more trustworthy than a single reading, for observations that are too large are likely to counterbalance others that are too small. Greater confidence can be placed in the mean when the separate readings differ but little from the mean than when they differ greatly. The average of the differences between the mean and the separate readings is called the *mean deviation.* It can be shown (as indicated in §9) that when ten observations are made, the probability that the actual error is greater than the mean deviation is very small, about 1 in 100, while if 15 observations are made it is reduced to 1 in 1000. Even if only 5 observations are made (which is rather too small a number) the probability is only 1 in 15. Hence, when a quantity is measured several times, the average deviation may be taken as a measure of the possible error.

6. Possible Error of a Calculated Result.

A piece of laboratory work usually calls for the measurement of several different quantities and the calculation of a result by some formula. Knowing the possible errors of the separate quantities we can deduce the possible error of the result, but the method will vary with the nature of the arithmetical operations.

(a) *Possible Error of a Sum or Difference.*—The possible error of a sum or difference is the sum of the possible errors of the separate quantities, for each possible error may be either positive or negative.

Example.—A bulb containing air (Exp. VI) weighs 20.1425 g. ±0.0002 g. and after the air has been pumped out it weighs 20.0105 g. ±0.0002 g. Hence the weight of the air is 0.1320 g. ±0.0004 g. Since it is sometimes erroneously assumed that a derived result must be accurate to as high a percentage as the measurements from which it is deduced, it should be noticed in the above that, while the separate weights are found to 0.001%, the weight of the air is only ascertained to 0.3%.

(b) *Possible Error of a Power.*—If a measured quantity x is in doubt by p per cent (p being small), the nth power of x is in doubt by np per cent. For

$$\left\{ x\left(1 \pm \frac{p}{100}\right) \right\}^n = x^n\left(1 \pm \frac{np}{100}\right)$$

squares and higher powers of $p/100$ being neglected.

Example of (a) and (b).

$T = 3.506 \pm .005$ and $t = 2.018 \pm .003$. (Exp. VII). What is the possible error of $T^2 - t^2$? $T^2 = 12.29$ and since T may be in error by 1/7 %, T^2 may be in error by 2/7 % or .04. Hence $T^2 = 12.29 \pm .04$. Similarly $t^2 = 4.07 \pm .01$. Hence $T^2 - t^2 = 8.22 \pm .05$.

(c) *Possible Error of a Product or Quotient.*—The percentage by which a product or quotient is in doubt is the sum of

the percentages by which the separate quantities are in doubt. For if the quantities be

$$x\left(1 \pm \frac{p}{100}\right) \text{ and } y\left(1 \pm \frac{q}{100}\right)$$

their product is

$$x\left(1 \pm \frac{p}{100}\right) \cdot y\left(1 \pm \frac{q}{100}\right) = xy\left(1 \pm \frac{p+q}{100}\right)$$

and their quotient is

$$x\left(1 \pm \frac{p}{100}\right) \Big/ y\left(1 \pm \frac{q}{100}\right) = \frac{x}{y}\left(1 \pm \frac{p+q}{100}\right)$$

$p/100$ and $q/100$ being assumed small. It is evident that a similar statement applies to any number of products and quotients.

Example of (*b*) *and* (*c*).

The diameter of a sphere (Exp. III) is measured by a vernier caliper and found to be 1.586 cm., but the vernier only reads to 1/50 mm.; so the possible error is .002 cm. or 1/8 of 1%. The sphere is weighed in a balance such that 1 mg. added to one pan does not cause an observable change of the pointer, while 2 mg. does, and the weight is, therefore, 16.344 g. with a possible error of .002 g. or 1/80 %. The calculated value of the density is 7.827; but the volume may be in error by 3/8 % and the mass by 1/80 %. Hence the density may be in error by 3/8 + 1/80 %, or practically 3/8 %. Hence the proper statement of the density is 7.83 with a possible error of .03 or $7.83 \pm .03$.

7. General Method for the Possible Error of a Result.

The above rules for sums, differences, powers, products, and quotients will usually suffice for finding the possible error of a result calculated from the measurements of several quantities. But when several of these operations are combined, or when the formula for calculation contains

one of the quantities more than once, the effects of the several errors may be difficult to trace by these means. The following general method is always applicable. It may be carried out by simple arithmetic, but is simplified by an elementary use of the calculus.

To find to what extent the possible error in one of the quantities affects the result, we may calculate the result assuming all the quantities to be quite accurate and then repeat the calculation after changing one of the quantities by its possible error. The difference in the result will be the effect sought. If we do the same for each of the other quantities, the final possible error of the result will be the sum (without regard to sign) of the parts due to the separate quantities.

This, however, is equivalent to differentiating the whole expression, first with regard to one quantity, then with regard to a second and so on and finally adding the partial differentials. It will be seen from the following examples that the process is much simplified by taking the logarithm of the whole formula before differentiating.

(1) *Time of Vibration of a Pendulum* (Exp. VII).—If in time T a pendulum makes n fewer vibrations than the pendulum of a clock that beats seconds and if t is the time of a single vibration,

$$t = \frac{T}{T-n}.$$

Taking logarithms,

$$log\ t = log\ T - log\ (T-n)$$

Hence by differentiating,

$$\frac{\delta t}{t} = \frac{\delta T}{T} - \frac{\delta T}{T-n}$$

$$= -\frac{n}{T(T-n)}\delta T.$$

This means that if T be changed by a small quantity, δT the consequent change, δt, in t is given by the formula. If

the possible error of T be 2 seconds, by putting $\delta T = \pm 2$ the value of δt will be the possible error of t. If T be 862 seconds and n be 17,

$$\frac{\delta t}{t} = \pm \frac{17 \times 2}{862 \times 845} = 0.005\%$$

This indicates one of the advantages of taking logarithms. It gives us at once the ratio of δt to t, or (multiplied by 100) the percentage by which t is in doubt.

(2) *Specific Heat by the Method of Mixture* (Exp. XIX).— Let $T = 95°$ be the initial temperature of the specimen, $t_0 = 25°$ that of the water, and let $t = 45°$ be the final temperature of the mixture, and let the possible error of each thermometer reading be 0.2°. The formula for calculation is

$$x = \frac{(m + m_1 s)(t - t_0)}{M(T - t)}.$$

We shall consider how far the possible errors in the thermometer readings affect x, leaving the consideration of the other terms (the errors of which are likely to be much smaller) to the reader.

$$\log x = \log (m + m_1 s) + \log (t - t_0) - \log M - \log (T - t)$$

Proceeding as in (1) above, we find the effects of the possible errors of T, t_0, and t respectively as follows:

$$\frac{\delta x}{x} = -\frac{\delta T}{T - t} = \frac{0.2}{50} = 0.4\%$$

$$\frac{\delta x}{x} = -\frac{\delta t_0}{t - t_0} = \frac{0.2}{20} = 1.0$$

$$\frac{\delta x}{x} = \frac{(T - t_0)\delta t}{(T - t)(t - t_0)} = \frac{70}{50 \times 20} 0.2 = 1.4$$

$$\text{Total} = 2.8\%$$

This example will show a second advantage in the method of taking logarithms. It separates the various terms and so simplifies the process.

8. Some General Notes on Errors.

The statement of a possible error should contain only one significant figure. (A zero that serves only to fix the decimal point, such as the zeros in 0.0026, is not a significant figure). Thus in the last example in §6, 3/8% of 7.83 is 0.0293, which shows that the second decimal place in 7.83 is in doubt by 3. Hence it would be superfluous to add figures to show that the third and fourth decimal places are also in doubt.

Measurements sometimes seem so accurate that one is tempted so say that "the possible error is practically zero and need not be considered." This is never literally true. One factor may be so accurately determined, compared with other factors, that the effect of its possible error on the result might seem to be negligible; but only a calculation can show this and the calculation will frequently show the opposite. An illustration occurs in the first example of §6, considered in connection with the other measurements required to determine the density of air in Exp. III.

The consideration of possible errors is of great importance in deciding what care need be expended in determining the various factors in a complex measurement and what are the best conditions for obtaining an accurate result. This applies more especially to advanced and difficult measurements, but illustrations will occur in this book. (Exp. XIX.) But, as one of the purposes of this course is to teach the most exact use of the measuring instruments, measurements should usually be made as accurately as the instruments will permit.

9. "Probable Error" of a Mean.

There is another method of indicating the reliability of measurements which possesses some advantages over the one that we have explained, though it is not so generally applicable. When a large number of observations of a quantity have been made, we can, by means of formulas

deduced from the mathematical Theory of Probability, calculate the probability that the mean is not in error by more than a given amount. When a coin is tossed up it is an even chance whether it will come down a head or a tail; the chance or probability of its being a head is, therefore, 1 in 2 or 1/2. Now the "probable error" of the mean of a number of readings is defined as a magnitude such that it is an even chance whether the error is greater or whether it is less than this magnitude. In other words, the probability of the error exceeding the "probable error" is 1/2. One formula for calculating the "probable error" is the following:

$$e = 0.84 \frac{\text{average deviation}}{\sqrt{\text{number of observations}}}.$$

This method is useful as a method of indicating the reliability of measurements when each of *all* the quantities that occur in the experiment can be measured several times. For when each mean and its "probable error" has been found we can calculate the "probable error" of the final result. For further details we shall refer the reader to other works (e. g., Merriman's "Least Squares"). We shall not have frequent occasion to refer to "probable errors," since in most cases some of the quantities that have to be determined cannot be measured more than once.

In justification of the use of the mean deviation as a measure of the possible error, we may note that by the above formula for e, when 10 observations have been made, the mean deviation equals $3.6\ e$. Now a reference to tables of the probability of errors (e. g., Smithsonian Tables) shows that the probability of an error greater than $3.8\ e$ is about 1 in 100.

10. Limits to Calculations.

By the above methods the possible error in any calculation from experimental quantities may be deduced. The magnitude of the possible error in any calculation indicates

how far it is useful and desirable to carry the calculation. *A calculation should be carried as far as, but not farther than, the first doubtful figure.* This rule must be applied not only to the calculation of the final result, but also to each intermediate step. When a calculation is carried too far, useless and very unscientific labor is expended, and when it is not carried far enough very absurd results are often obtained.

In addition and subtraction a place of decimals that is doubtful in any one of the quantities is doubtful in the result.

In multiplication and division (performed in the ordinary way) decimal places that have not been determined are usually filled up by zeros. Any figure in the result that would be altered by changing one of these zeros to 5 is doubtful.

11. Notation of Very Large and Very Small Numbers.

Partly to save space and partly to indicate at once the magnitude of very large or very small numbers, the following notation is used. The digits are written down and a decimal point placed after the first and its position in the scale indicated by multiplying by some power of 10. Thus 42140000 is written 4.214×10^{7} and .00000588 is written 5.88×10^{-6}. This also enables us to abbreviate the multiplication and division of such numbers. Thus $42140000 \times .00000588$ is the same as $4.214 \times 5.88 \times 10$ and $42140000 \div .00000588$ is the same as $(4.214 \div 5.88) \times 10^{13}$.

12. Plotting of Curves.

It is assumed that the general method of the representation of the connection between two related quantities by means of a curve is familiar to the reader from his work in Graphical Algebra or elsewhere. Attention may be called to the following points:

1. Mark experimental points clearly by crosses or circles surrounding the points.

2. The curve should be drawn so as to strike an average path among the points; *it does not have to pass through even one point.*

3. Abscissas may be drawn to any scale and ordinates to any scale. Record the main division of the scales along each axis. (Do not record the individual observations.)

4. For convenience, all the abscissas may be diminished by the same amount before plotting, and the same is true of ordinates.

5. Ordinates and abscissas should be drawn to such scales that the curve occupies a large part of the paper.

6. Curves should be drawn carefully and neatly by means of curve forms.

MECHANICS.

13. The Use of a Vernier.

The *vernier* is a contrivance for reading to fractions of the units in which scales are graduated. It is a second scale parallel to the main scale of the instrument and so divided that n of its units equal $n-1$ or $n+1$ of the units of the scale. If s is the length of a scale unit and v that of a vernier unit, in the first case

$$n\,v=(n-1)s \text{ or } s-v=s\div n;$$

in the second case

$$n\,v=(n+1)s \text{ or } v-s=s\div n.$$

Hence the unit of the vernier is less or greater than that of the scale by one-nth of a scale unit.

If we did not have a vernier there would be something in the nature of an index to indicate what division of the scale should be read in making a certain measurement and fractions would be estimated by eye. The zero of the vernier is taken as such an index, the whole number of scale divisions being the number just below the zero of the vernier, while the fraction of a scale division is determined with the vernier. If the mth division of the vernier coincides with the scale division, the zero of the vernier must be m $n$ths of a scale unit from the scale division just below it. Thus the use of a vernier divided into n parts is equivalent to subdividing the scale unit into n parts.

FIG. 1.

If no vernier division exactly coincides with a scale division there will be two vernier divisions nearly coincident

with scale divisions, and one can often estimate fractions of the fraction given by the vernier. In figure 1 the whole number of scale divisions is 5.2, and evidently the third and fourth vernier divisions most nearly coincide with scale divisions. Since the third division is somewhat nearer a coincidence, we may call the fraction 3.3 tenths, or, the full reading will be 5.233.

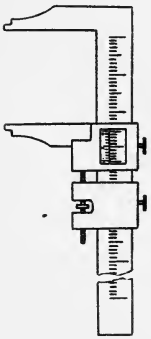

FIG. 2.

14. Vernier Caliper.

The *vernier caliper* consists of a straight graduated bar, and two jaws at right angles to it, one of which is fixed while the other is movable. The position of the movable jaw can be accurately determined by means of the scale and a vernier which should read zero when the jaws are in contact. (If this be not the case, allowance must be made for the zero reading).

15. Micrometer Caliper.

The *micrometer caliper* is a U-shaped piece of metal in one arm of which is a steel plug with a carefully planed face and through the other arm of which passes a screw with a plane end parallel and opposite to that of the screw. A linear scale on the frame reads zero approximately when the plug and screw are in contact, and its reading in any other position indicates the whole number of turns of the screw and consequently the number of mm. (or 1/2 mm. or 1/40 in. as the pitch of the screw may be) between the screw and the plug. Fractions of a turn are read on the divided head. As contact approaches, the screw should be turned with a very light touch and the same force used for different contacts. Some micrometers are provided with a ratchet head which permits only a definite, moderate pressure.

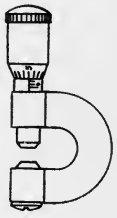

FIG. 3.

16. Micrometer Microscope.

The *micrometer microscope* is a microscope with cross-hairs at the focus. In one type of instrument these cross-hairs are movable by a micrometer screw. In the other and more common type the whole microscope is moved by a micrometer screw (see Fig. 4). The most elaborate instruments have both movements. The rotations of the screw are read on a fixed linear scale while the fraction of a rotation is read by a circular scale attached to the screw, and thus the amount of movement is ascertained if the pitch of the screw is known. The pitch is best determined by reading a length on a reliable scale placed in the field of view.

17. Comparator.

The *comparator* consists essentially of a pair of microscopes movable along a horizontal bar to which they are at right angles. The length to be measured is placed under the

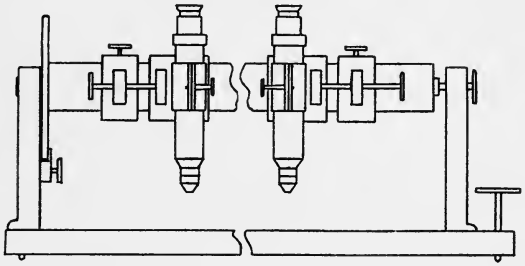

FIG. 4.

microscopes. The eye-piece of each microscope is first focused clearly on the cross-hairs and the whole microscope focused *without parallax* on the point to be observed, so that the image of the point coincides with the intersection of the cross-hairs. The object is then removed and a good scale put in its place, and a reading of the scale gives the required length, this reading being facilitated by the use of micrometer screws.

18. Spherometer.

The *spherometer* is an instrument with four legs, three of which form the vertices of an equilateral triangle, while the fourth is at the center of the triangle. The fourth leg can be screwed up and down and the distance of its extremity from the plane of the extremities of the other three legs can be accurately measured by means of a linear scale attached to the fixed legs and a circular scale attached to the movable leg. The linear scale gives the number of complete turns of the screw and the circular scale the fraction of a turn. These scales are read when the screw makes contact with an object placed beneath.

The *position of contact* may be determined by noticing that the screw turns very easily for a fraction of a turn just after contact begins. This is due to reduced friction in the bearing, owing to the weight of the screw falling on the body in contact and to the back-lash of the screw, the frame not yet being raised. The screw should be lowered until it thus begins to turn very easily; it should then be turned back again until it just again begins to turn hard and a reading then made. A less sensitive method is to turn the screw down until the instrument is felt to rock or wobble and make a reading; then raise the screw until rocking just ceases, make another reading and take the mean of the two as the contact reading. In some spherometers the end of the screw on making contact raises two levers arranged to greatly magnify the motion. The screw is lowered until the top lever comes to some definite position, for instance, with the end opposite to a stud in the frame of the instrument.

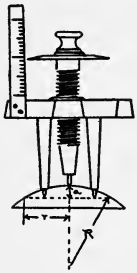

FIG. 5.

The *zero reading* is the reading of the scales when the end of the screw is in the plane of the ends of the legs, and is so called because the instrument is most frequently used to determine distances above or below this plane. It may be

obtained by placing the instrument on a very plane plate of glass. Several zero readings should be made before and after making readings with the object in position under the screw, for the zero reading is likely to change from slight disturbances of the adjustment of the instrument and thermal expansion due to the heat of the hand. If the mean of the zero readings made after reading on an object should be decidedly different from the mean of those made before, readings should be again made on the object and the first zero readings discarded. After each reading, the screw should be turned up through at least a quarter revolution, that the readings may be entirely independent.

The *unit of the linear scale* may be obtained by comparison with a standard steel scale. If the plane of the circular scale is not exactly perpendicular to the axis, the linear scale may not give the correct number of turns. It is therefore best, as a check upon the linear scale readings, to count the number of rotations.

19. The Dividing Engine.

The *dividing engine* received its name from its being originally made to subdivide scales, diffraction-gratings, etc. An equally frequent use of the instrument is for the accurate measurement and calibration of scales, gratings, etc.

It consists essentially of (1) a very carefully made horizontal screw, the ends of which are so supported that the screw is free to rotate, but not to advance or recede; (2) a nut movable on the screw and bearing against (3) a platform movable along a track which is parallel to the screw; (4) a micrometer microscope which in some instruments is held in a support movable along a rail on the same bed plate as the track, but in other instruments is carried on the platform; (5) dividing gear for making scales, etc.; (6) a divided circular scale attached to the screw with a vernier attached to the bed plate.

If, by means of the circular scale and vernier, the rotation of the screw can be read to 1 in a very large number, say in 1 in 1,000, then, since the nut moves a distance equal to the "pitch" of the screw (measured parallel to the axis) when the screw is given one complete rotation, it follows that the movement of the nut and platform can be read to a correspondingly small fraction of the pitch of the screw.

The object whose length is to be measured is placed on the movable platform (in the case of instruments of the first type mentioned under (4) above). The microscope is focused on one end of the length to be measured, so that the intersection of the cross-hair coincides with that end. By turning the screw until the movement of the platform brings the other end of the length to be measured into coincidence with the intersection of the cross-hairs and observing the number of turns and parts of a turn, the length of the object in terms of the pitch of the screw as unit is obtained. The true pitch of the screw must itself be obtained by comparing it by the same method with some accurately known length, such as a length on a standard meter.

The adjustment of the microscope consists of two steps: (1) the eye-piece must be focused on the cross-hairs (but the eye-piece must not be taken out lest the cross-hairs be injured); (2) the whole microscope must be moved toward or away from the object until it is seen *without parallax*, i. e., until the relative position of the cross-hairs and the image of the object is not changed by shifting the eye sidewise. The length to be measured must then be placed parallel to the screw. This is attained when, by rotation of the screw, the image of each end can be brought to coincidence with the intersection of the cross-hairs. One of the most frequent sources of error, in using a measuring instrument on the screw principle, is back-lash or lost motion. To avoid this the screw should always be turned in the same direction during a measurement. In many dividing engines back-lash is impossible because the motion of the

screw cannot be reversed, the platform can only be moved in the reverse direction by unclasping the nut (which, for this purpose, is a split nut held together by a clasp and spring).

Usually the handle for turning the screw is not attached to the screw or circular scale, but to a separate disk rotating co-axially with the screw. The motion of the handle *in one direction* is communicated to the screw by a ratchet; when the handle is reversed the ratchet slips freely. As an aid to counting the number of turns of the screw in measuring a considerable length, two detents are sometimes geared to the screw in such a way that only a definite number of turns can be given to the screw at a time, after which the handle must be turned back for beginning a new number of turns.

A more complete description of the dividing engine will be found in *Stewart and Gee*, I, §16.

20. Cathetometer.

The *cathetometer* is a vertical pillar supported on a tripod and leveling screws, and capable of rotation about its axis; the pillar is graduated and a horizontal telescope with cross-hairs is borne by a carriage that travels on the pillar and can be clamped at any desired position. A slow-motion screw serves for accurate adjustment of the position of the telescope.

Adjustments.* (1) *The intersection of the cross-hairs, X, must be in the optical axis of the telescope.* To secure this, focus X on some mark, rotate the telescope about its own axis and see whether X remains on the mark. If not, the adjusting screws of the cross-hairs must be changed until this is attained.

(2) *The level must be properly adjusted.* Level the telescope until the bubble comes to the center of the scale. Turn the level end for end. If the bubble does not come to

*Adjustments (1) and (2) are not usually required and should not be made without the advice of the instructor.

the same position, the level must be adjusted until it will stand this test.

(3) *The scale must be vertical.* If there are separate levels for the shaft, this is readily attained. If there is but one level for telescope and shaft, this and the next adjustment must be made simultaneously.

(4) *The telescope must be perpendicular to the scale.* The top of the scale, T, may be regarded as having two degrees of freedom—first, parallel to the line of two leveling screws of the base, A and B; second, in a line through the third leveling screw, C, perpendicular to AB. If A and B be screwed equal amounts in opposite directions, T will move parallel to AB. If C only be turned, T will move perpendicular to AB.

First make the telescope horizontal and parallel to AB. Turn the shaft through 180°. It is easily seen that if the telescope makes an angle a with the normal to the scale, turning the scale through 180° will cause the telescope to make an angle $2a$ with its former direction. Hence, with the leveling screw of the telescope, correct half the error in the level, and, by turning A and B equally in opposite directions, correct the remainder. Turn the telescope to the first position and repeat the above adjustments, then to the second and continue as often as is necessary. Then turn the telescope normal to AB and adjust by C. When the adjustment is complete, turning the shaft through any angle will not alter the position of the bubble.

Unless the cathetometer is on a perfectly immovable support, perfect adjustment is not possible and too much time should not be spent in adjusting, provided the telescope is accurately level at each reading.

The eye-piece of the telescope is focused (but not removed) until the cross-hairs seem perfectly distinct and the focus of the objective changed until the object is seen very distinctly and without parallax, i. e., with no relative motion with respect to the cross-hairs when the eye is moved about.

21. Barometer.

Text-book of Physics (Duff), pp. 157–158; *Watson's Physics*, pp. 150–154; *Ames' General Physics*, pp. 176–177; *Crew's Physics*, pp. 165–166.

If the barometer is of *Fortin's cistern form*, the cistern is raised or lowered by means of the screw at the bottom until the mercury just meets an ivory stud near the side of the cistern. A collar to which is attached a vernier is so placed that the top of the meniscus of the mercury column is tangent to the plane of the two lower edges. The height of the barometer should be reduced to zero by the formula

$$h_0 = h(1 - .000162t)$$

where h is the observed height, h_0 the height at 0° and t the temperature Centigrade. For the expansion of the mercury will increase the height in the ratio $(1. + .000181t)$ and the expansion of the brass scale will reduce the apparent height in the ratio $(1 - .000019t)$ (Table VII).

The *siphon barometer* has two scales, graduated on the glass tube, in opposite directions, from a common zero. The length of the mercury column is obviously the sum of the readings of the mercury levels in the two tubes. Since the coefficient of linear expansion of glass is only about 0.000008, the correction formula becomes

$$h_0 = h(1 - .000173t).$$

Since the mercury may adhere to the glass to some extent, *barometer tubes should be tapped gently before reading.*

22. The Balance.

Kohlrausch, §§7–11; *Watson's Practical Physics*, §§25, 26.

Weighing by a Sensitive Balance.—On first using a sensitive balance note the position, purpose, and structure of the following parts:—

The beam,	The knife-edges and planes,
The pointer,	The arrestment,
The pillar,	The rider-arms,

The pan-supports.

By the sensibility of a balance is meant the amount of deflection of the beam produced by a given small weight. Consider how the sensibility depends on (1) the length of the beam, (2) the weight of the beam and pans, (3) the distance of the point of suspension of the beam from its center of gravity. Will the sensibility of a certain balance be different with different loads on the pans and why? How can the sensibility be varied with a given load? (See references.)

Precautions in Use of Balance.

1. Note the maximum load that may be placed on the balance and take care not to exceed it.

2. Always lift the beam from the knife edges before in any way altering the load on the pans.

3. Do not stop the swinging of the balance with a jerk. It is best to stop it when the pointer is vertical.

4. To set the beam in vibration, do not touch it with the hand, but raise and lower the arrestment.

5. Place the large weights in the center of the pan.

6. Make final weighings with the case closed.

7. Replace all weights in their proper place in the box when they are not actually in use. Do not use weights from different boxes.

8. Do not place anything in contact with a pan that is liable to injure it.

9. Avoid, if possible, weighing a hot body.

10. Never handle the weights with the fingers, as this may change some of the weights appreciably. Always use the pincers.

Notice the dimensions of *the weights* in the box, e. g., 50 g., 20 g., 10 g., 10 g., 5 g., 2 g., 1 g., 1 g., etc. Instead of weights of 0.005 g., 0.002 g., 0.001 g., 0.001 g., it is customary to use a rider of 0.01 g., which can be placed on the beam at various distances from the center. The beam is for this purpose graduated into 10 divisions, which may be still further subdivided. Thus the 0.010 g. rider placed at the division 4 of the beam is equivalent to 0.004 g. placed on the pan.

The *zero-point* of the balance is the position on the scale behind the pointer at which, the pans being empty, the pointer would ultimately come to rest; it must not be confused with the zero of the scale. As much time would be wasted in always waiting for the pointer to come to rest, the zero of the balance is best obtained from the swings of the pointer. For this purpose, readings of the successive "turning-points" are made as follows—three successive turning-points on the right and the two intermediate ones on the left, or vice versa; e. g.,

Turning points.

	L.	R.
	−1.3	+2.1
	−1.1	+2.0
	−1.0	
Mean,	−1.13	+2.05
		−1.13
Zero-point =		+0.92 ÷ 2 = +0.46.

By taking an odd number of successive turning-points on one side and the intermediate even number on the other side and then averaging each set, we eliminate the effect of the gradual decrease of amplitude of the swing.

The *resting-point* of the balance with any loads on the pans is the point at which the pointer would ultimately come to rest, and is found in the same way as the zero-point. If the resting-point should happen to be the same as the zero-point, the weight of the body on one pan is immediately found by the weights on the other pan and the position of the rider. Usually, however, this will not be so. With the rider at a suitable division, find the resting-point on one side of the zero-point, and then, after altering the rider one place, find the resting-point on the other side of the zero. By interpolation the change of the position of the rider necessary to make the resting-point coincide with the zero-point is deduced. For example, the

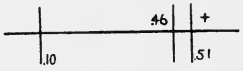

FIG. 6.

zero is +0.46; with the rider at 4 the resting-point is +0.51; with the rider at 5 the resting-point is +0.10. By changing the rider from 4 to 5, 0.001 g. was added. To bring the resting-point to the zero we should have added $0.05 \div (0.51 - 0.10)$ of 0.001 g. or 0.0001 g. approximately. Hence the weight of the body is the weight on the pan plus 0.0041 g.

The arms of the balance may be unequal. If this be so, the weight obtained above will not be the true weight. To eliminate this error the body must be changed to the other pan and another weighing made. If l be the length of the left arm and r that of the right and if u be the counterbalancing weight when the body is in the left pan and v when it is in the right, while w is the true weight of the body then,

$$lw = ru,\ lv = rw$$
$$\therefore w = \sqrt{uv} = \tfrac{1}{2}(u+v).$$

(The geometric mean of two very nearly equal quantities is nearly equal to their arithmetic mean.) The ratio of the arms of the balance may also be calculated, since

$$\frac{l}{r} = \sqrt{\frac{u}{v}}.$$

The buoyancy of the air on the weights and on the body must be allowed for in accurate work. To the apparent weight of the body must be added a correction equal to the weight of the air displaced by the body and from the apparent weight must be subtracted the weight of the air displaced by the weights. In each case the weight of the air displaced can be calculated if its volume and density are known. This correction in any case is very small. A small *percentage* error in the correction will not appreciably affect the calculated true weight. Hence approximate values of the volumes of the body and weights may be used. In finding the volume of the weights the density of brass weights may be taken as 8.4. The density of air at 0° and 760 mm. may be taken as .0013, and its density at the temperature of the laboratory and the pressure indicated

by the barometer may be calculated by the laws of gases. Hence the temperature and barometric pressure should be obtained.

23. Adjustment of Telescope and Scale.

To adjust a telescope and scale, determine approximately the location of the normal to the mirror, either by finding the image of one eye or the image of an incandescent lamp held near the eye. Move the stand supporting the telescope and scale until the center of the scale is about in line with the normal. Look along the *outside* of the telescope at the mirror and move the scale up and down, or, if this is not possible, raise or lower the stand until you see the reflection of the scale in the mirror. It may be a help to illuminate the scale with an incandescent lamp. Look through the telescope pointed at the mirror, and change the focus until the scale is seen distinctly. Remember that the more distant the object, the more the eye-piece must be pushed in, and that the image of the scale is at about twice the distance of the mirror.

24. Time Signals.

A convenient source of time signals for a laboratory is a chronometer which either opens or closes a circuit containing batteries, sounders, etc., every second with an omission at the end of each minute.

The individual second intervals indicated by a chronometer, so arranged, are likely to be somewhat inaccurate, and therefore, when an accurate interval of one second is required, a second's pendulum should be used with a platinum point making contact with a drop of mercury, and thus, if desired, closing an electric circuit. Since it is difficult to set the mercury drop exactly in the center of the path, alternate seconds are likely to be too long. Therefore, if possible, a two seconds' interval should be employed, alternate contacts being disregarded. If these contacts cause confusion, a pendulum omitting alternate contacts may be used (see *Ames and Bliss*, p. 486).

I. TO MAKE AND CALIBRATE A SCALE.

To illustrate the use of the dividing engine (described on page 17) a short scale is to be engraved in millimeters on a strip of nickel-plated steel and then calibrated by comparison with the average millimeter of a standard scale.

Arrange the cogs of the dividing gear so that each fifth mm. division shall be longer than the intermediate divisions and each tenth division still longer. Test this adjustment on a rough test strip. Next clamp the strip to be divided on the platform of the engine so that it is parallel to the screw; this can be tested by observing the edge of the strip in the microscope as the platform is advanced by the screw. Care should be taken to clamp the pillar that supports the divider so that the point of the divider moves perpendicular to the length of the scale. A scale of 2 or 3 cms. should then be marked out on the steel strip and the temperature of the platform ascertained by a thermometer.

This scale is next to be calibrated. The exact pitch of the screw is first obtained in terms of the mm. of the standard. For this purpose a considerable length, e. g., a decimeter, of the standard should be measured on the engine. This should be done for three different parts of the screw. The agreement of the three determinations will afford some indication of the uniformity of the screw. The scale should then be measured mm. by mm. For the first reading the circular scale of the screw may be set to zero when the cross-hairs coincide with the zero division of the scale to be measured, and thereafter the screw should be turned always in the same direction and only arrested for a reading of the circular scale and vernier (the total number of turns being also noted) when the microscope shows that the middle of a division has come to coincide with the intersection of

the cross-hairs. As this coincidence approaches, the handle should be turned slowly, and if turned too far the reading at that point must be omitted altogether. The handle should also be turned slowly when contact with the detent approaches so that the screw may not be arrested with a jerk.

As a check on the work, the whole length of the scale should be measured.

In calculating the true length of the divisions, allowance must be made for the temperature of the standard which may be taken as the temperature of the platform of the engine. The standard is correct at the temperature marked. From its coefficient of expansion calculate the length of its mm. at the temperature of observation and then deduce the pitch of the screw at the same temperature. Then from the readings made, calculate the length of each millimeter of the scale and, by addition, draw up a table showing the true distance of each division from the zero division.

Questions.

1. Enumerate the possible sources of error in the use of the dividing engine for the manufacture of scales.
2. At what temperature would the whole length of your scale be an exact number of centimeters? (Table VII.)

II. ERRORS OF WEIGHTS.

Kohlrausch, §12; *Watson's Practical Physics*, §27.

Weights by good makers are usually so accurate that errors in them may for most purposes be neglected. But when less perfect weights are to be used or when weighings are to be made with the highest possible degree of accuracy, the errors in the weights must be carefully ascertained.

We shall suppose that a 100 g. box of weights is to be tested, and that a reliable 100 g. weight is supplied as a standard, and that an accurate 10 mg. rider is supplied for making the weighings. The weights of the box will be denoted by 100′, 50′, 20′, 20″, 10′, and so on, and the

sum $5' + 2' + 2'' + 1'$ by $10''$. To find the six unknown quantities, $100'$, $50'$, $20'$, $20''$, $10'$, $10''$, we must make six weighings and obtain six relations between these quantities. Such a set of weighings are indicated in the following table. Each should be performed by the method of double-weighing described on page 24.

$$\begin{aligned} 10' &= 10'' + a \\ 20' &= 10' + 10'' + b \\ 20'' &= 20' + c \\ 50' &= 20' + 20'' + 10' + d \\ 100' &= 50' + 20' + 20'' + 10' + e \\ 100 &= 100' + f \end{aligned}$$

To solve these equations, substitute the value of $10'$ given by the first in the second; then substitute the value of $20'$ given by the second in the third, and so on to the last, when the value of $10''$ in terms of the standard 100 and a, b, c, d, e, f will be obtained. The calculation of the other quantities will then present no difficulty. To standardize the box completely the same process must be applied to $10'$, $5'$, $2'$, $2''$, $1'$, $1''$, and similarly to the smaller weights.

III. VOLUME, MASS, AND DENSITY OF A REGULAR SOLID.

The mass of the specimen (a sphere or cylinder) is found by weighing on a sensitive balance (see p. 21). To eliminate the inequality of the arms of the balance, the body should be weighed in both pans (p. 24). The zero-point and resting-points of the balance should be found by the method of vibrations and the various precautions in the use of the balance must be carefully observed. Allowance should be made for air buoyancy (p. 24) and corrections should be applied to the weights, if the weights have been corrected in the preceding experiment, or if a table of corrections is supplied.

The dimensions of the specimen are measured by a micrometer caliper (p. 14) or a vernier caliper (p. 14). If the body is spherical, ten measurements of the diameter should be made and the average taken; if it is cylindrical ten measurements of the diameter and ten of the length should be made.

From the mass and the volume, the density (or mass per c.c.) is deduced.

The ratio of the arms of the balance should also be derived from the results of the double weighing (p. 24).

The possible error of the density determination should be calculated as illustrated on p. 6.

Questions.

1. If the object aimed at were merely the density of the body, which of the above measurements should be improved in precision and to what extent would it need to be improved?
2. If the above improvement were not possible, how much of the refinement of measurement of the other quantity might be discarded?

IV. MOHR-WESTPHAL SPECIFIC GRAVITY BALANCE.

Kohlrausch, p. 45; *Stewart and Gee*, I, §92, III.

This is a convenient form of hydrostatic balance for finding the density of a liquid by determining the buoyancy of the liquid on a float hung from an arm of the balance and immersed in the liquid. Instead of weights riders are used, the arm of the balance from which the float hangs being graduated into ten divisions. The float is made of such a size that when hanging in air from the graduated arm of the balance (which is less massive than the other arm) it will just produce equilibrium. Four riders of different mass are employed, each one being ten times as heavy as the next smaller. The largest rider is of such a size that if the float hanging from the balance be immersed in water at 15° C. the addition of the rider to the hook at the

end of the beam will restore equilibrium. Hence it counterbalances the buoyancy of the water on the float. Thus it is evident that if the water be replaced by a liquid of unknown density at the same temperature (so that the volume of the float is the same) and if the largest rider under the circumstances produces equilibrium when placed at the sixth division, then for equal volumes, this liquid can weigh only six-tenths as much as water, or its density is 0.6. A second rider, one-tenth as heavy as the first, would evidently

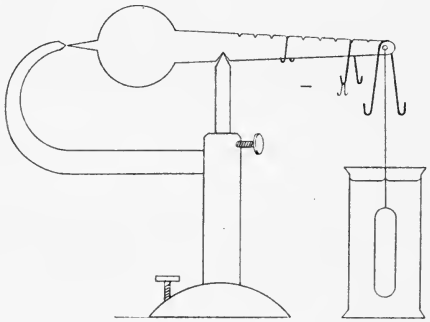

FIG. 7.

enable us to carry the process one decimal place farther, etc. For liquids of a density exceeding unity, another rider equal to the largest must be hung from the end of the beam, and still a third may be necessary for liquids of density above 2.

From the above it will be seen that (1) the balance must be adjusted by the leveling screw on the base until the end of the beam is opposite the stud in the framework when the float is suspended in the air; (2) the beaker must always be filled to the same level, that level being such that when the liquid is water at 15° C. the balance is in equilibrium with the largest rider hanging above the float, and (3) the liquid tested must be at 15° C.

As an exercise in the use of this balance, find what shrinkage of volume there is in the solution of some salt (e. g., common salt, ammonium chloride or copper sulphate) in water and find how the shrinkage varies with the concentration. Solutions may be made up by weighing out very carefully on a sensitive balance (see p. 21), 0.5 gm., 1 gm., 4 gm., 10 gm., etc., of the salt and dissolving each in a deciliter of water. When the density of a solution has been found, the percentage contraction is calculated from the sum of the volumes of the constituents before mixture and the volume of the solution after mixture; the volume in each case equals the mass divided by the density. The densities of various salts are given in Table VII.

The densities found and the percentages of contraction should be represented by curves with percentages of salt as abscissæ. If any determination of density be largely in error it will be shown by the curve.

If time permit, determine the density at 15° of equivalent solutions* of several salts having the same base, e. g., NaCl; 1/2 Na_2SO_4; $NaNO_3$; etc., and compare with the densities of similar solutions with a different base, e. g., NH_4Cl; 1/2 $(NH_4)_2SO_4$, NH_4NO_3, etc. The difference in density between corresponding salts should be approximately constant (*Valson's Law of Moduli*). Find similarly the difference in densities contributed by the acid radicals, e. g., $NaNO_3$ and NaCl; NH_4NO_3 and NH_4Cl, etc.

Questions.

1. What sources of error may there be in a determination of density by this method?
2. How might the accuracy of the riders be tested?
3. How might the accuracy of graduation of the beam be tested?
4. What effect has capillarity?
5. Explain the Law of Moduli.†

* The chemical equivalent of a substance is the atomic or molecular weight divided by the valency. Two solutions are equivalent if the number of grams of each dissolved in one liter (or that proportion) is the same fraction of the respective chemical equivalent.

† *Phy. Chem.*, Ewell, p. 159.

V. DENSITY BY VOLUMENOMETER.

Gray's Treatise on Physics, I, §426.

When the density of such substances as gunpowder, sugar, starch, etc., is to be determined, neither the method of immersion in a liquid nor that of the direct measurement of mass and volume can be employed. The method then usually employed is that of the *volumenometer*. This is a method of immersion in air instead of immersion in water, with an application of Boyle's Law instead of Archimedes' principle. The volume of the body is found by placing it in a glass vessel and noting how much the pressure in the vessel changes when the air is allowed to expand.

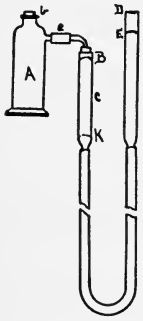

FIG. 8.

A gas-washing bottle of about 150 c.c. capacity, A, into which the body is to be introduced, is connected, by heavy pressure tubing, with an open, U-shaped, mercury manometer (see Fig. 8). The bottle, A, is closed by the stopper b, which should be lubricated with rubber grease,* and forced into A to a definite mark. DE is raised until the mercury in the burette BC is at a division B which is carefully observed. The pressure, P, in A is carefully determined from the difference in mercury levels and the barometer. By the use of a rear mirror, parallax may be avoided and a small square will assist in reading a scale between the two arms of the manometer. The accuracy of the readings may be increased by using a cathetometer (p. 19). Lower DE until the mercury is at a division K and again determine the pressure, p. Let the volume between B and K be v. Let V be the volume of A, and connecting tubing, to B. By Boyle's Law:

$$PV = p(V + v)$$

*Equal parts pure rubber gum, vaseline, and paraffin. The two latter are melted together and the rubber is cut into small pieces and dissolved in the heated liquid.

Make at least six determinations of P and p, bringing the mercury each time to the same points B and K which should be as far apart as is convenient. Calculate V from the mean values.

Now introduce a carefully weighed amount of the assigned powder into the bottle, A (which may be disconnected at e), and insert the stopper, b, to its former depth. Again determine the pressures, P' and p' when the mercury level is at B and K respectively. Repeat as before. If x is the unknown volume of the powder, the previous equation becomes

$$P'(V-x)=p'(V-x+v)$$

from which x may be calculated. From the volume and mass of the powder its density is determined.

If time permit, determine the density also with a specific gravity flask (pyknometer). Weighings should be made of (1) bottle empty; (2) bottle filled with a liquid of known density which is inert toward the body, and (3) with a known mass of the body in it, the rest of the bottle being filled with the liquid. An equation for density can be worked out.

The possible error in the determination of the density is found by methods explained on pages 3, 8.

Questions.

1. What sources of error remain uneliminated?
2. With a view to greater accuracy what suggestions would you make as to the most suitable magnitudes for x and v?

VI. DENSITY OF AIR.

The *density of air* at atmospheric pressure, or its mass per cubic centimeter, might be obtained by weighing a flask containing air at atmospheric pressure and then re-weighing it after all the air has been removed by an air-pump. The difference of weight, together with the volume of the flask, would give the density of the air. In practice the

procedure has to be modified, because it is impossible to completely exhaust the flask of air. The modification consists in finding the pressure of the air remaining in the flask and taking account of it.

Let D be the required density at the room temperature and pressure, P. Let d be the density of the remaining air when the pressure has been reduced to p. Let the weight of the flask when filled with air be W and let w represent its weight when exhausted to the pressure p.

$$W-w=V(D-d)$$

By Boyle's Law

$$\frac{D}{d}=\frac{P}{p} \quad \therefore \quad \frac{D-d}{D}=\frac{P-p}{P}$$

Therefore

$$D=(D-d)\frac{P}{P-p}=\frac{W-w}{V}\cdot\frac{P}{P-p}.$$

A convenient form of flask is a round-bottom flask from which part of the neck has been cut off and which is closed by a rubber stopper containing a glass tube with a glass stop-cock. The rubber stopper will hold tighter if lubricated with rubber grease * before insertion.

If the flask, as found, is dry, it will be better to postpone finding its volume until the end of the experiment, as the operation requires it to be filled with water. Moreover, of the two weighings for finding the mass of air removed, it is better to make the one with the flask partly exhausted first, for the weighing with the air admitted can be made immediately after, without handling the flask or removing it from the balance, a point of some importance where the difference of weight to be measured is so small. To save delay in weighing the flask after it has been exhausted, the zero reading of the fine balance used should be obtained before the flask is exhausted. For the method of accurate weighing, by oscillations, see page 23.

*See note, p. 32.

A Bunsen's aspirator or a Geryk pump is satisfactory for exhausting the flask. The flask should be connected to the aspirator or pump, through a bottle for catching any water or mercury. An open-tube manometer connected to the tube that joins the aspirator or pump and flask will give the pressure.

There should be a stop-cock or a rubber pinch-cock in the connection between the manometer and the pump or aspirator. When a sufficiently high exhaustion has been secured this cock should be closed for several minutes to ascertain if there is any leakage. If not, both ends of the manometer should be read and the stop-cock of the flask closed. Before removal of the flask, the other cock should be opened that the rest of the apparatus may fill with air. If by any chance a small quantity of water should pass into the manometer, allowance should be made for it, the density of mercury being taken as 13.6.

The flask is then weighed as quickly as possible on a fine balance, the method of vibration being used. It may be necessary to hang the flask by a fine wire to the hook which carries the pan. This weighing is repeated with the stop-cock open, but with the flask otherwise undisturbed. The atmospheric pressure is obtained from a reading of the barometer (see p. 21).

The volume of the flask may be obtained by filling it with distilled water and weighing it on an open balance. To get the flask just filled to the stop-cock, the stopper (removed for filling the flask) should be thrust in with the stop-cock open, the stop-cock should then be closed, and any water above the stop-cock should be removed. Of course, the stop-cock should be replaced at its original depth, which should be marked. The density of water at different temperatures will be found in Table V.

When the experiment is completed, place the open flask inverted on a frame to dry, so that it may be ready for the next person who uses it.

The density of dry air may be found in the same way,

the flask being several times exhausted and refilled through a drying-tube. Similarly the density of any other gas, e. g., carbon dioxide, may be found by filling the flask from a generator. The gas must be admitted to the exhausted flask very slowly and the exhaustion and filling must be repeated to insure the (almost) complete removal of the air.

In reporting, deduce from your measurement of the density of air or gas, its density at 0° C. and 760 mm. by using Boyle's and Charles' Laws. Find also the possible error of the measurement of density (p. 5).

Questions.

1. Would the first results be affected by the presence of water in the flask? Explain.
2. Should the flask weigh more filled with dry air or filled with moist air, both at atmospheric pressure? Why?

VII. ACCELERATION OF GRAVITY BY PENDULUM.

Text-book of Physics (*Duff*), §117; *Watson's Physics*, §§112–114; *Watson's Practical Physics*, §§46–49; *Ames' General Physics*, pp. 74, 91, 135; *Crew's Physics*, §§85, 86.

The *acceleration of gravity*, g, is most readily obtained from the length and time of vibration of a pendulum. The time of vibration of an ideal simple pendulum, i. e., a heavy particle vibrating at the end of a massless cord would be

$$t = 2\pi\sqrt{\frac{l}{g}}$$

l being the length of the pendulum. If the bob is a ball so large that the mass of the suspending wire is negligible, the above formula will apply provided the radius of the ball is negligible compared with the length of the pendulum. If these assumptions may not be made, the pendulum must be regarded as a physical pendulum and its moment of

inertia about the suspension considered. Under these circumstances the formula

$$t = 2\pi\sqrt{\frac{I}{Mgh}}$$

must be used, where I is the moment of inertia of the entire pendulum about the knife-edge, M is the total mass and h is the distance from the knife-edge to the center of gravity of the whole. If the mass of the suspension is negligible it is only necessary to consider the moment of inertia of the ball about the knife-edge. It is easily shown that the latter formula then reduces to the formula for the simple pendulum, provided the length of the pendulum is taken as the distance from the knife-edge to the center of the ball plus $2r^2/5l$ where r is the radius of the ball. Hence to find g there are three quantities, t, l, and r, to be measured.

A convenient form of pendulum consists of a spherical bob into which screws a nipple through which a fine wire is passed and secured. To the upper end of the wire is soldered a stirrup of brass which rests on a knife-edge of steel. A short platinum wire should be soldered to the lower side of the bob.

For accurately *measuring the length* of the pendulum a *cathetometer* (see p. 19), which should be carefully adjusted, may be used. (If necessary, the measurement of length may be postponed until the time has been observed). The horizontal cross-hair of the cathetometer is first focused on the knife-edge, the fine screw being used for the final adjustment of the telescope, and the scale and vernier are then read. The telescope is then lowered and set on either the top or bottom of the bob, whichever is the more definite. These readings should be repeated several times, beginning each time with the knife-edge. If the adjustments are imperfect, the telescope should at least be made exactly level before each reading. The *diameter of the bob* may be

measured by means of a *micrometer* or a *vernier caliper* (see p. 14).

For fixing the vertical position of the pendulum, two vertical pointers may be so placed that, when the pendulum is at rest, the pendulum suspension and two pointers are in one plane. The eye of the observer should always be kept in this plane in using the first two methods. The pendulum is set vibrating in an arc of 3 or 4 cms. Several attempts may be necessary to get the pendulum vibrating exactly perpendicular to the knife-edge with the bob free from rotation.

The time of vibration is most readily obtained with precision when the pendulum is very nearly a second's pendulum, i. e., when the period of a complete vibration is very nearly two seconds. For the determination of the period several methods are available. The first and roughest method given below will serve for adjusting the pendulum to the required length.

(A) In the first method for determining the *period*, time is found by the relay (p. 25) and the number of vibrations in three minutes is counted, fractions of a vibration being roughly estimated. This is repeated several times. Or a stop-watch or stop-clock may be used, but it should be rated by comparison with a chronometer or standard clock. The stop-watch is started as the pendulum crosses the plane of observation and "one" is counted the next time the pendulum crosses the plane in the same direction. The watch is stopped on the 50th vibration, and the whole repeated five times. The mean time divided by 50 will give a fair value for the period.

(B) A second and much more accurate method of obtaining the time of vibration is the *method of coincidences*. This consists in finding the rate at which the pendulum gains or loses as compared with a standard clock or chronometer. It is applicable only when the periods of pendulum and clock or chronometer are nearly the same or when one is nearly an exact multiple of the other. The method receives its

name from the fact that what is observed is the "coincidence interval" or the interval between the moment when a passage of the pendulum through the vertical coincides with some signal from the clock to the next time when such a coincidence occurs.

In a coincidence interval, the pendulum must gain or lose one vibration as compared with the chronometer or other time standard. If n such coincidence intervals occur in T sec., the number of vibrations of the pendulum during this time is $(T \pm n)$. Hence if t is the time of one vibration,

$$t = \frac{T}{T \pm n},$$

and the period of a complete vibration is

$$t = \frac{2T}{T \pm n}.$$

A convenient form of signal is given by the chronometer and relay described on page 25. It is advisable to have the coincidence interval something between 30 seconds and 3 minutes, and, if necessary, the length of the pendulum should be changed for the purpose.

After the coincidence interval has been roughly determined by a few observations, the following modification of the method will give it much more accurately. Calling the time of the first coincidence zero seconds, observe the second on which the next coincidence occurs and then the next, until four have been observed. Then, after allowing a considerable number of coincidences to pass unnoted, but keeping note of the time, observe the number of the seconds, counted from the original coincidence, upon which four more successive coincidences occur.

From the first set of coincidences, three estimates of the coincidence interval will be obtained and three others from the second set, the mean of all giving an approximate estimate. Then let the time of the first coincidence of the first set be subtracted from the time of the first of the second

set, also the time of the second coincidence of the first set from that of the second of the second set, etc. These differences give four estimates of the time, T; of some unknown integral number, n, of coincidence intervals. If the mean of these four estimates be divided by the mean time of a single coincidence interval as already found, the quotient will be n plus or minus a small fraction. This fraction is due to inaccuracy in the estimates of the coincidence intervals and should be dropped. The period t of the pendulum may now be calculated. The plus sign in the denominator is used if the pendulum is the faster.

The following aid to the observation of coincidences is suggested. Keeping the eye constantly in the proper plane for observation, make a dot on a piece of paper at each click of the relay. When there appears to be coincidence, prolong the dot into a stroke. To avoid recording every click, a cross may be used instead of a dot for marking a minute, and the clicks may be passed unrecorded until the next minute, or coincidence. There may be several successive clicks during which there appear to be coincidences, in which case several successive strokes should be made and the mean taken. From these dots, strokes, and crosses, the times of coincidence may be deduced. Or, a dial indicating seconds may be employed, the second when there first appears to be a coincidence being observed and the second when there first appears to be no coincidence. Since the clock cannot be observed immediately, the ticks are counted until the clock is observed and then subtracted; minutes must be noted and recorded if they are not recorded on the clock.

(*C*) A third method consists in modifying the second method so that coincidences of two sounds are observed. The pendulum is made to actuate a sounder or telephone each time it passes through the vertical and a coincidence is observed when the sounder and relay strike together. A block of wood with a narrow trough filled full of mercury is placed in a mercury tray and is adjusted beneath the pendulum so that the platinum wire on the under side of the

bob just touches the mercury when the pendulum is at rest, and crosses the narrow trough at right angles when the pendulum is in motion,. A wire soldered to the knife-edge is connected in series with several batteries, a sounder or telephone, and the mercury trough. The final adjustment of the mercury trough is made with the leveling screws of the mercury tray. Care should be taken not to spill the mercury.

From the possible errors in the measurements of l and t deduce the possible errors in the value found for g (see p. 7).

Questions.

1. Does the friction of the knife-edges and of the air increase or decrease the value of g?
2. Why should coincidence be observed exactly for the plane containing the position of rest?
3. What would be the result of increasing the arc of vibration to 10 cm.? (Table III.)
4. Why should the top reading of the cathetometer always precede the bottom reading?
5. Design, if possible, a scheme of electrical connections such that the sounder will only operate when there is a coincidence.

VIII. COEFFICIENT OF FRICTION.

Text-book of Physics (*Duff*), §§126–130; *Watson's Physics*, §§96–100; *Ames' General Physics*, p. 118; *Crew's Physics*, §117; *Daniell's Physics*, pp. 176–184.

The *coefficient of friction* of two surfaces is the ratio of the force of friction opposing the incipient or actual relative motion to the force pressing the two surfaces together. The force requisite to start the motion is greater than that required to sustain the motion, i. e., the "coefficient of static friction" is greater than that of "kinetic friction." Moreover, the coefficient of kinetic friction is not quite constant, but varies somewhat with the speed.

(A) The *coefficient of static friction* of one surface on another may be found by means of a block of the former resting on a slide of the latter. One end of the slide is gently

elevated by a screw until the block just fails to stand stationary on the slide. The tangent of the angle which the slide then makes with the horizontal equals the *coefficient of static friction* (see references). The tangent may be measured by some simple method, using meter-stick, plumb-line and level or square. Several entirely independent adjustments for this angle and measurements of the tangent should be made, the adjusting screw being each time turned some distance down so that the influence of the previous setting may be avoided. The friction may vary somewhat from point to point, and if so, different points should be chosen for the separate trials.

The accuracy of the determination of the tangent should be calculated to see whether the possible errors will account for the variations of the coefficient. Such, however, will probably not be found to be the case.

(B) The *coefficient of kinetic friction* may be determined by the same apparatus if we can find the acceleration with which the block moves down the slide when the latter is tilted beyond the angle of repose. For, if the acceleration of the block is a and its mass m and the angle of inclination of the slide i, then the component of gravity down the slide is $mg \sin i$ and the pressure on the slide is $mg \cos i$. Hence, if μ is the coefficient of friction, by Newton's second law,

$$m\,a = m\,g \sin i - \mu\, m\, g \cos i$$

$$\text{and, } \mu = \tan i - \frac{a}{g \cos i}.$$

This process will give the mean coefficient of friction for the range of speeds through which the block passes, but for the low speeds in question the coefficient does not vary much.

The *acceleration*, a, is found by a method frequently employed in physical measurements. A tuning-fork (frequency of 50 or less) is fastened in a clamp attached to a support above the slide. A stylus of spring brass with a steel needle point is attached to one prong and just behind this stylus is a second stationary stylus which is attached to

the support. A long and narrow glass plate is covered with the washing compound called "Bon Ami" by transferring, with a wet cloth, a little of the paste from the cake to the glass, and then spreading it out in a thin layer. The block is then raised to the top of the slide and secured by a trigger. The support that carries the fork is raised and lowered and the fork is adjusted in the clamp until each stylus touches the coated glass, making with it an angle of about 45°, the stylus on the fork being exactly in front of the other stylus.

The frame-work is then lifted until neither stylus touches the glass. The fork is set in vibration by drawing the prongs together with the fingers and releasing them, or by withdrawing a wooden wedge, and is then adjusted until its stylus vibrates an equal amount on each side of the other (stationary) stylus. The frame-work is then lowered until the styli touch the glass and the block is immediately released by the trigger.

A wave line should be obtained with a straight line exactly in the center, the amplitude of the wave line on each side of the straight line being several millimeters. Since in any measurement the effect of inaccuracies at the ends is less important the greater the quantity measured, we measure the distance passed over during several vibrations of the fork. This distance divided by the time in which it was traversed, i. e., by the period of the fork multiplied by the number of vibrations, gives the average velocity of the block during this time. If T be the period of the fork and x the distance passed over in n complete vibrations of the fork, the average velocity is x/nT. Similarly we find the average velocity for the next n complete vibrations. The average acceleration will be the difference between these average velocities divided by their separation in time or nT; for since each velocity is the average we may consider it as belonging to the middle of the time for which it is the average. From several successive groups of n vibrations several values of the acceleration are obtained and the mean taken.

It remains to determine the *period of the fork.* Two methods will be described. (a) The fork is clamped beside a small electro-magnet connected through a battery with a pendulum which closes the circuit every second (see p. 25). To the armature of the electro-magnet a stylus is also attached. A plate of glass covered with "Bon Ami" is clamped on a movable block so that each stylus rests upon it. The electro-magnet and fork may have any relative position which may be convenient, but the styli should not be far apart. The fork is set vibrating and the block with the glass is drawn along, the fork making a wave line and the other stylus a straight line broken (or notched) every second. With a square, lines are drawn at right angles to the glass through the beginnings of alternate second signals and the number of complete vibrations, estimated to tenths, is counted between the lines.

(*b*) This is known as a *stroboscopic method* and depends upon the persistence of vision. The fork is watched through holes in a disk revolving at a constant speed. The holes are equally spaced in concentric circles, the number per circle increasing with the radius. The speed of the disk is varied until the fork appears stationary when viewed through the holes of a particular circle. If there are m holes in the circle, and if the disk revolves n times per second, the frequency of the fork is obviously mn. By varying the speed and using other holes, additional determinations may be made. The speed of the disk is obtained by determining, with a counter, the number of revolutions in a given time.

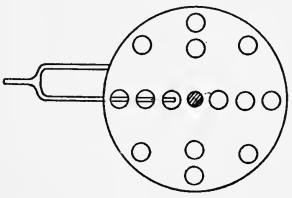

FIG. 9.

(C) Another method of finding the coefficient of kinetic friction is to make the slide horizontal and find the force required to keep the block in uniform motion after it has been started. For this purpose a braided cord is attached to one end of the block, passed over a pulley at the end of

the slide, and attached to a scale pan, to which weights are added. In this case the weight of the pan and weights must not be taken as the force acting on the block, for some force is required to overcome the friction of the pulley. The amount required must be found by a separate experiment. Two pans are attached to the ends of the cord hanging over the pulley and sufficient equal weights are placed on the pans to make the pressure on the pulley the same as in the main experiment where the parts of the cord were *at right angles*. The additional weight on one scale pan requisite to keep the whole in constant motion when started is the force needed to overcome the friction of the pulley, and is, therefore, the correction required.

With this apparatus we may also test whether the cœfficient of friction varies when weights are added to the block. The correction for friction of the pulley does not need to be re-determined experimentally, but may be calculated from the former determination, on the assumption that the friction of the pulley is proportional to the pressure on it.

The possible error in the results of the first and last methods is easily determined. The most accurate way of finding the possible error in method (B) is by means of formulæ deduced by the differential calculus (see p. 6), but a much simpler and a sufficiently accurate method is the following: Note that an overestimate of i will increase the value of μ and the same will be the effect of an underestimate of a. Hence the coefficient should be recalculated with tan i and cos i increased and a decreased by their possible errors and the change found in the coefficient may be taken as the final possible error. The possible error of a may be taken as its mean deviation and the possible errors of tan i and cos i may be deduced from the measurements from which they were obtained.

Questions.

1. In the second method, why is it desirable that the straight line be exactly in the middle of the wave line?
2. In the third method, what error would be introduced if the cord from the block was not exactly horizontal?

IX. HOOKE'S LAW AND YOUNG'S MODULUS.

Text-book of Physics (*Duff*), §§168, 171, 173; *Watson's Physics*, §§172, 173; *Ames' General Physics*, pp. 144, 145, 153, 154; *Crew's Physics*, §§126–129.

Hooke's Law states that, for small strains, stress and strain are proportional. *Young's Modulus*, E, is the constant ratio of stress to strain for a stretching strain, the stress being taken as the force per unit cross section and the strain as the stretch per unit of length, or, if F is the whole force, A the area of cross section, L the whole length, and l the increment of length,

$$E = \frac{F}{A} \div \frac{l}{L} = \frac{FL}{Al}.$$

(A) The quantity most difficult to measure is l, the small increase of length. If a wire be supported at one end and force applied to the other end, there is danger that the support may yield slightly, and a slight amount of yielding will cause a proportionally large error in the estimate of the small increase in length. The peculiarity of the first method described below is the means adopted to eliminate the yield of the support. The increase of the length of the wire under experiment is found by comparison with another wire under constant stretch attached to the same support as the former wire. One wire carries a scale and the other a vernier opposite the scale. If there be any doubt which is vernier (see p. 13) and which is scale, comparison should be made with an ordinary steel scale. The screws by means of which the wires are clamped to scale and vernier should be adjusted until scale and vernier tend to lie in one plane. A light rubber band may then be slipped over scale and vernier to keep them together.

FIG. 10.

The stretch may be produced by means of lead weights. The value of these weights should be determined by a

platform balance. To produce a suitable stretch it may be advisable to add two or more weights at a time. We shall suppose that two are added, but the description can readily be modified to suit any number. The greatest weight should not be more than half that required to break the wire. (A copper wire 0.01 sq. cm. section will break at 40 kgs.; brass, 60 kgs.; iron, 60 kgs.) Suppose, then, two weights are added at a time and each stretch observed. When the maximum number has been added the weights should be removed in the same order, readings being again taken as they are removed. The whole series of observations should be repeated at least three times. Such readings should always be arranged in tables having in a line or column all the readings for a particular pair of weights. The length of the wire may be measured by means of a long beam compass and the diameter should be measured at least a dozen times at different places and in different directions by means of a micrometer caliper (see p. 14).

Before calculating, the dimensions should be expressed in centimeters and the weights in dynes. First find the mean value of l for each pair of weights when added and when removed and then the value of $F \div l$ for each of these values of l and the respective F's. Find the mean value of $F \div l$ and the greatest percentage deviation from the mean. This will give the percentage deviation from Hooke's Law since $F \div l$ should be a constant, A and L being practically constant. The final value of Young's Modulus should be stated in the notation explained on page 11.

The possible errors of the different quantities measured may be taken as the mean deviation in each case. The percentage error of the final value of E will be, as is readily seen from the formula, the sum of the percentage errors of F, L, l, and twice the percentage error of the radius (see p. 6).

(B) Young's Modulus may also be found by means of the *flexure* of a bar. For, in bending (within limits) one side of a bar is stretched and the other compressed (negatively stretched), and so Young's Modulus is the only constant

that need be considered. The amount of bending might be deduced from the sag of the center or end of the bar, but a much more delicate method is the following optical one:

A bar of rectangular cross section is laid on two knife-edges and at each end is attached an approximately vertical mirror in mountings that admit of considerable adjustment. A vertical scale, nearly in line with the bar, is reflected from the farther mirror into the nearer and thence into a telescope also nearly in line with the bar. When a weight is attached to the center of the bar, the bar is bent and another part of the scale is reflected into the telescope. This arrangement serves to determine the angle of bending. For suppose the difference of the scale-readings on the horizontal cross-hairs of the telescope be D cms. (Fig. 11) and let the distance between the two mirrors be p and the distance of the scale from the farther mirror q, then, if the change of inclination of each mirror be i,

$$\tan i = \frac{D}{2p + 4q}.$$

For a consideration of figure 11 will show that $d_1 = p \tan 2i$; $d_2 = q \tan 4i$. But since i is a small angle $\tan 2i = 2 \tan i$ and $\tan 4i = 4 \tan i$.

$$\therefore D = d_1 + d_2 = (2p + 4q) \tan i.$$

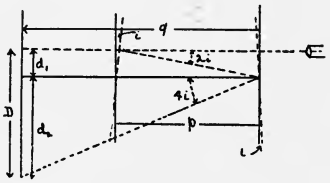

FIG. 11.

From $\tan i$, the weight R in dynes applied at the center, the length of the bar between the knife edges, l, the breadth, b, and the thickness, a, Young's Modulus, E, is obtained, by the equation

$$E = \tfrac{3}{4} \frac{Rl^2}{a^3 b \tan i}.$$

PROOF.

Let the y axis coincide with the radius from the center of curvature of the bar to the center of the bar, and let the x axis be the tangent to the central axis at this point. Designate distances from the elastic central axis, LOM (Fig. 12), along other radii by z. The elastic central axis remains unchanged in length. The curvature at any point P on LOM is the rate of change of the directions of the tangent. The angle the tangent line at P makes with the x-axis is a small one and may be taken as dy/dx (which is really the tangent of that angle). The rate of change of the direction of the curve at the point x, y, is therefore d^2y/dx^2, which therefore equals the curvature. But the curvature also equals $1/r$, r being the radius of curvature. Hence

$$\frac{1}{r}=\frac{d^2y}{dx^2}.$$

Now consider two strips of the beam distant $\pm z$ from LOM. By the bending these strips are changed in length in the proportion z/r or $z\ d^2y/dx^2$. (For, consider figure 13; the proportional change of length is

$$\frac{G'H'-GH}{GH}=\frac{G'C-GC}{GC}=\frac{GG'}{GC}=\frac{z}{r}\Big).$$

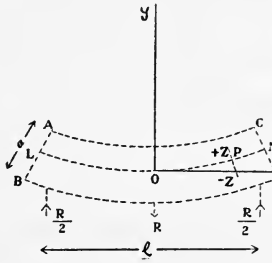

FIG. 12.

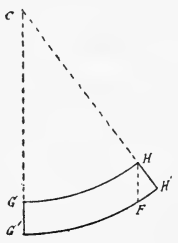

FIG. 13.

By definition of Young's Modulus, if a force F applied to a rod of cross section A and length L produce an extension l,

$$F=\frac{EAl}{L},$$

where E is Young's Modulus. The stress in a strip of width b and thickness dz is obtained by putting bdz for A and $z\ d^2y/dx^2$ for $l\div L$, which gives

$$Ebdz.z\frac{d^2y}{dx^2}.$$

Hence the moment about P of the restoring force in the strips $\pm z$ is

$$2Ebdz\ .z^2\frac{d^2y}{dx^2}.$$

The moment about P of the stress in the whole cross section is the integral with reference to z of the above expression for values of z from 0 to $\frac{1}{2}a$ or

$$Eb\frac{a^3}{12}\frac{d^2y}{dx^2}.$$

For equilibrium this must equal the moment of $\frac{1}{2}R$ about P or $\frac{1}{2}R\,(\frac{1}{2}l-x)$

$$\therefore \frac{d^2y}{dx^2}=\frac{6R}{Ea^3b}\left\{\frac{l}{2}-x\right\}$$

$$\therefore \frac{dy}{dx}=\frac{6R}{Ea^3b}\left\{\frac{lx}{2}-\frac{x^2}{2}\right\}$$

At a point of support

$$x=\tfrac{1}{2}l;\ \frac{dy}{dx}=\tan i.$$

Hence by substitution

$$E=\tfrac{3}{4}\frac{Rl^2}{a^3b\tan i}.$$

(By integrating again, the value of y at a point of support or the deflection of the beam is obtained. This is left as an exercise for the student.)

The adjustment of the apparatus is most readily made as follows. Place the telescope and scale nearly in the line of the mirrors and, glancing above the telescope, set the farther mirror so that the nearer mirror is seen by reflection and then the latter so that the scale is seen. Then adjust the eye-piece of the telescope so that the cross-hairs are as distinct as possible and finally focus the telescope until the scale is seen. The bar must not be strained beyond the limits of elasticity. For adjustment of the telescope and scale, see p. 25. Equal weights, perhaps 100 grams at a time, should be added, but this process should be stopped when it is found that the scale-reading no longer changes in the same proportion as the weights. Determine carefully by several readings, with and without this maximum weight attached, the change of scale-reading. The width and thickness of the bar may be measured by a micrometer caliper (see p. 14), a number of readings of each at different points being made.

In calculating, use this weight for R and the average of the changes of deflection for D and take the mean deviation

as the measure of the possible error of D, a, and b. The percentage possible error of $\tan i$ is deduced from the possible errors of D and $2p+4q$. The possible error in the latter term is twice the possible error in p plus four times that in q.

(C) A simple optical method may also be employed for finding the extension of a wire. In this method, one side of a small bench carrying a vertical mirror is supported by the end of the wire and the other by a fixed bracket. The deflection of the mirror when weights are added to the wire is read by a scale and telescope. The details of the method may readily be worked out by anyone who has followed the preceding methods.

(D) (Searle's Method.) The extension may also be determined from the change of position of a level supported by the two wires. The lowering of the stretched wire is compensated by a micrometer screw which therefore reads the extension. For details, see *Watson's Practical Physics*, §45.

Questions.

1. How closely is it worth while to measure the length of the wire in the first method?
2. Which of the first two methods is the more accurate and what is the chief weakness of the other?
3. In the second method why is nothing said as to the distance of the mirrors beyond the knife-edges? Might they be placed inside?
4. Reduce your results to tons and inches.

X. THE RIGIDITY (OR SHEAR-MODULUS).

Text-book of Physics (*Duff*), §§119, 170; *Watson's Physics*, §171, 174, 175; *Ames' General Physics*, pp. 151–153; *Crew's Physics*, §§131, 132; *Duff's Mechanics*, §§117, 130, 131.

The *rigidity* of any material is the resistance it offers to change of shape without change of volume. It is measured by the ratio of the shearing stress to the shear produced. In the twisting of a wire or rod, within moderate limits, there is no change of volume. Hence this affords a means of finding the rigidity of the material. The *constant* or *modulus of torsion* of a particular wire is the couple

required to twist one end of unit length of the wire through unit angle, the other end being kept fixed. If it be denoted by τ and if the length of the wire be L the couple required to twist the wire through unit angle is τ/L. If now to the wire be attached a mass of moment of inertia, I, and the wire and the mass be set into torsional vibrations, the time of a semi-vibration is, by the principles of Simple Harmonic Motion (see references),

$$t=\pi\sqrt{\frac{IL}{\tau}}.$$

If t, L and I be found τ can be deduced. From the modulus of torsion of the particular wire the rigidity n of the material of which the wire consists can be deduced; for

$$n=\frac{2\tau}{\pi R^4}.$$

Proof.

Suppose unit length of the wire to be twisted through unit angle. The vibrations are due to the restoring couple at the lower end produced by the twist. Let the cross section of the end be divided into concentric rings and let the radius of one ring be x and its width dx; its area is $2\pi x dx$. Relatively to the fixed end it is displaced through unit angle. Hence the linear displacement (supposed small) of the ring whose radius is x is x times unit angle or simply x. This is by definition the shear and hence the shearing stress is nx. This is the restoring force per unit area of the cross section. Hence the restoring force of the ring whose radius is x is $2\pi nx^2dx$. The effect of this force in producing rotation depends on its moment about the axis or $2\pi nx^3dx$. The moment of the restoring force of the whole end section is the sum of expressions like $2\pi nx^3dx$ for values of x between o and r or $\frac{1}{2}\pi nr^4$. This is by definition the modulus of torsion, τ and gives us the above equation. It should be noticed that it is not a constant for the material of the wire, but depends on the dimensions of the particular wire.

The *length*, L, may be measured by means of a long beam compass which is afterward compared with a fixed brass scale. The radius, R, may be measured by a micrometer caliper (see p. 14), measurements being made at a great many different places and the mean taken.

The *moment of inertia*, I, of the wire and attached mass

might be roughly obtained by calculation, but it is better to apply an experimental method that is used in other cases. This consists in adding to the vibrating mass, of unknown moment of inertia, another mass of such regular form that its moment of inertia can be accurately calculated, and finding the times of vibration before (t) and after (T) adding this mass. If the original moment of inertia be I and the added moment of inertia i:

$$\frac{T}{t}=\frac{\sqrt{I+i}}{\sqrt{I}},$$

whence

$$I=i\frac{t^2}{T^2-t^2}.$$

One of the simplest forms of added inertia is that of a solid cylinder of circular cross section vibrating about an axis through the center of the axis of the cylinder and at right angles to it. The vibrating mass may then be in the form of a hollow cylinder in which the solid cylinder may be placed. If l be the length and r the radius of the solid cylinder of mass m:

$$i=m\left(\frac{l^2}{12}+\frac{r^2}{4}\right).$$

The quantities l and r can be obtained with sufficient precision by measurement with a steel scale divided to mm.'s, and m may be found by a platform balance. In the above formula for i, it is assumed that the axis of rotation is perpendicular to the axis of the cylinder. That this may be so the carrier must be carefully leveled. This may be done by supporting close under it a rod that is carefully leveled by a spirit-level and comparing the carrier as it swings with the leveled rod. The end of the vibrating mass should be provided with an index, such as a vertical needle. A stationary vertical wire is placed in front of this index when the latter is in the position of rest. The body is set

vibrating through an angle of between 60° and 90°, all pendulum vibrations being carefully suppressed.

The *time of vibration* may be found by much more accurate methods than simply timing a certain number of vibrations. The most common methods for accurately timing vibrations are the "method of coincidences" and the "method of passages." The former is especially useful for finding the time of vibration of a pendulum whose half period is approximately one second (Exp. VII). The method of passages will be found suitable for the present experiment. It consists in noting as accurately as possible the time of every nth passage of the vibrating system through its mean position or position of rest. The value to be chosen for n is a matter of convenience when two observers work together, one counting the seconds and the other noting the passages, or when a single observer has a chronometer in front of him. But a single observer noting time by a clock circuit and sounder should choose for n an odd number such that n semi-vibrations occupy a little more than a minute. (It is supposed that there is a minute signal, such as the omission of a tick; see page 25.)

The passages are observed as follows: After a minute signal, the seconds are counted until a passage occurs, for example, from left to right. The second and fraction of a second of this passage is recorded. The succeeding passages in each direction are counted until the minute signal, after which the seconds are again counted until the passage occurs from right to left, for which the second and fraction of a second is recorded. Obviously, if n has been properly chosen, the passage just recorded is the nth. The succeeding passages are counted until the next minute signal, after which the second and fraction of a second of the $2n$th vibration (from left to right) is recorded.

The following suggestion may aid in counting the seconds and estimating fractions of a second. The observer should keep counting seconds (not necessarily out loud) along with the clock; when the number of the second is of two or

more syllables, the accent should be thrown on one syllable whose sound should coincide with the tick; thus, *elev*en, *thir*teen, *four*teen, etc., twenty-*one*, twenty-*two*, etc. The passage will usually occur somewhere between two ticks. To estimate at what point of time between the two seconds the passage takes place, the indications of the eye may be used to reinforce those of the ear. Suppose A (in Fig. 14) to be the mean position of the index on the vibrating body, then if B and C be its positions at the fifth and sixth ticks, respectively, and if BA be six-tenths of the distance BC, it is evident that the true time of passage is 5.6 seconds. With practice the eye can become very expert in making such judgments, and, for the purpose of attaining such skill, the method should be used from the beginning, although at first not much reliance can be placed on the judgment.

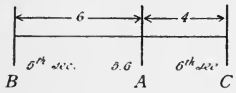

FIG. 14.

For simplicity of description we shall suppose that n is 5, but the proper substitutions must be made if n has any other value. When the approximate time of 5 vibrations has been obtained by observing a few passages, all of the passages need not thereafter be observed in order to ascertain when each fifth passage is due, for this can readily be foreseen by adding to the time of the last observed passage the known approximate time of 5 vibrations. Further assistance is obtained by recording the time of the 0, 10th, 20th, etc., in a second column, the first column being headed "left to right" and the second "right to left." In this way such a record as the following is obtained:

———L. to R.———		———R. to L.———	
M. S.	M. S.	M. S.	M. S.
(0)	(50)	(5)....	(55) ....
(10)	(60)	(15)	(65)
(20)	(70)	(25)	(75)
(30)	(80)	(35)	(85)
(40)	(90)	(45)	(95)

from which the time of vibration is calculated thus:

M. S.	M. S.
(50) — (0)—....	(55) — (5)—....
(60) — (10)	(65) — (15)
(70) — (20)	(75) — (25)
(80) — (30)	(85) — (35)
(90) — (40)	(95) — (45)
Mean of 50 vibrations—....	—....

Final mean of 50 vibrations = ...possible error....

Final mean of one vibration = ...possible error....

To find the possible error in the value found for n, first eliminate τ and i from the equation given above and express n in terms of the quantities observed L, l, T, t R, m. ($r^2/4$ is so small compared with $l^2/12$ that the effect of the possible error in the former may be neglected.) T and t come in only in the form $(T^2 - t^2)$ and the possible error in this may be found by methods stated on page 5.

Questions.

1. To increase the accuracy of the result, which quantity would have to be measured more closely?
2. What sources of error are there other than those referred to in the text?

XI. VISCOSITY.

Text-book of Physics (*Duff*), §§196–198; *Watson's Physics*, §161; *Ames' General Physics*, pp. 139, 168.

A solid has rigidity; that is, it offers a continued resistance to forces tending to change its shape. A liquid has no rigidity and offers no continued resistance to forces tending to change its shape; that is, the smallest force if given time will produce an unlimited change in the shape of the liquid. But the *rate* at which a liquid changes its shape under a given force is not the same for all liquids. Some liquids change very slowly and are called viscous liquids, others change rapidly and are called mobile liquids. The

action of both can be stated in terms of a property called *viscosity*.

The viscosity of a fluid may be defined as the ratio of the shearing stress in the fluid to the rate of shear. From this general definition a simpler definition can be readily deduced. A shear consists essentially in the sliding of layer over layer and the shearing is the force per unit area required to produce the shear. Hence we have the following equivalent definition: "The coefficient of viscosity is the tangential force per unit of area of either of two horizontal planes at unit distance apart, one of which is fixed while the other moves with unit velocity, the space between the two being filled with the liquid." (Maxwell.)

(A) The flow of liquid through a capillary tube is essentially of the nature of sliding of layer over layer. The cylindrical layer in immediate contact with the tube remains fixed or at least has no motion parallel to the axis of the tube, and the immediately adjacent layer slides over it, the next layer slides over the second, and so on up to the center of the tube. (In a tube of greater than capillary bore this is not so, for there are eddies in the motion. This distinction is in fact the best definition of the term capillary.)

Thus, if we measure the force causing flow through the tube and the rate of flow, we shall be in a position to deduce the coefficient of viscosity of the fluid. In fact, if M be the mass of liquid of density d that flows in time t, through a vertical tube of length l and radius of bore r, and if h be the vertical distance from the level of the liquid in the reservoir above the tube to the lower end of the tube, the coefficient of viscosity is

$$\eta = \frac{\pi g d^2 h r^4 t}{8Ml}.$$

Proof.

Suppose all the liquid in a capillary tube of length l and radius r to be solidified except a tubular layer of mean radius x and thickness dx. If there be a difference of pressure p (per unit of area) between the two ends, the solid will attain a steady velocity such that the

viscous resistance just equals the whole difference of pressure on its ends. Hence it follows from the definition of the coefficient of viscosity that

$$\frac{2\pi x l v \eta}{dx} = \pi x^2 p. \quad \text{Hence, } v = \frac{p x dx}{2 l \eta}.$$

If q be the volume of the core that flows out per second,

$$q = \pi x^2 v = \frac{p \pi x^3 dx}{2 l \eta}.$$

Suppose now another layer liquefied. There will follow a further flow represented by the same expression but with a different value for x. Let the process be continued until the whole is liquid, then the whole flow per second, Q, will be the sum of all the values of q for values of x between o and r. Hence

$$Q = \frac{p \pi r^4}{8 l \eta}.$$

If the tube be vertical and the flow be due to gravity only, instead of p we must put gdh. If M be the mass of density d that flows out in time t,

$$M = Qdt \quad \therefore \quad \eta = \frac{\pi g d^2 h r^4 t}{8 M l}.$$

In the above it was tacitly assumed that the liquid adheres to the tube without any slip. If there were any slip the outflow would be increased by it and the above expression would not hold. Poiseuille and others verified the above formula in all cases, thus showing that no slip occurs. (A more formal proof of the above equation is given in Tait's "Properties of Matter," §317).

A piece of capillary tubing should be chosen whose bore is as nearly as possible circular in section. This can be tested by examining the ends under a *micrometer microscope* (see p. 15). If the section is found to be nearly circular the principal diameters of the bore should be measured. This should, however, only be regarded as a preliminary measurement, serving as a test of the circularity of the bore and a check on the following more satisfactory method.

The mean radius of the bore can be best determined by weighing the amount of mercury that fills a measured length of the tube. For this purpose the tube should be first cleaned by attaching it to the end of a rubber tube, at the other end of which is a hollow rubber ball, and thus drawing through it and forcing out a number of times (1) chromic acid; (2) distilled water; (3) alcohol, and finally drying it by sucking air through it. Then draw into the tube

a column of clean mercury and measure its length as accurately as possible by a comparator (see p. 15).

The mass of the mercury should next be ascertained by weighing it with great care in a sensitive balance (for full directions see pp. 21–24). The mercury should not be dropped directly on the scale pan, but into a watch-glass or paper box placed on the scale pan. From these measurements and the density of mercury at the temperature of observation (see Table VII) the diameter of the bore is obtained. It may be noted that since it is r^4 that is used in the formula for viscosity and r^2 that is obtained directly from the mercury measurements of the bore the value of r need not be deduced. The length of the tube may be measured by the comparator as already described.

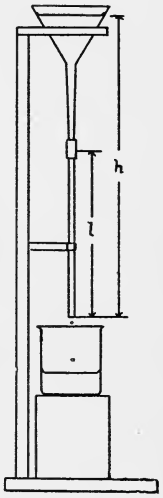

FIG. 15.

The tube is then attached vertically by a rubber connection to a funnel and the mass of water that flows through the tube in a given time found by weighing a beaker (1) empty and (2) containing the water that has passed. The time is obtained by observing a clock ticking seconds or a chronometer. It is evident that the greater the whole time the less the percentage error in time due to errors in observing the time of starting and stopping, and so, too, the greater the whole mass the less the percentage error in weight. Hence the time and the mass should be sufficiently great to make the percentage errors in them less than those in l and r^4. To prevent evaporation from the beaker it should be covered by a sheet of paper pierced by a hole through which the tube passes. While the liquid is flowing the temperature of the water in the funnel should be noted.

The value of h is the mean of its values at the beginning and end of the flow. These values are best obtained by a cathetometer (p. 19). For this purpose a very simple form

of instrument may be used. A vertical scale is placed near the apparatus for viscosity and the cathetometer (a telescope that may be leveled, movable along a vertical column that may be made truly vertical) placed so that its telescope (leveled to horizontality) may be turned, so that the intersection of its cross-hairs coincides alternately with the image of the water surface and that of the scale. This gives the level of the surface of the liquid on the vertical scale. The level of the lower end of the vertical tube is obtained in the same way, whence h is obtained.

The viscosity of alcohol may be measured by the same means, particular care being taken to prevent evaporation. The possible error of the result is readily calculated from the possible errors of the separate measurements. The possible error of r is not needed, but that of r^4 should be deduced directly from the determination of r^2.

Questions.

1. Could the radius be found satisfactorily by measurements with a micrometer microscope? Explain.
2. What mass of this liquid would flow through a tube 1 mm. in diameter and 1 meter long, under a constant head of 2 meters?
3. Two square flat plates of 20 cm. edge are separated by 1 mm. of this liquid. What force would be required to move one with a velocity of 30 cm. per second, the other being at rest?

XII. SURFACE TENSION.

Text-book of Physics (Duff), §§206–214; *Watson's Physics*, §§155–160; *Ames' General Physics*, pp. 182–190; *Crew's Physics*, §§149–160; *Poynting and Thomson, Properties of Matter*, Chap. XIV.

The height to which liquid rises or is depressed in a capillary tube depends on the surface tension of the liquid, the angle of capillarity, and the radius of the tube. From measurements of the height, h, and radius, r, the surface tension is deduced if the angle of capillarity is known, for (see references)

$$T = \frac{rdgh}{2 \cos \alpha},$$

d being the density of the liquid and g the acceleration of gravity. In the case of perfectly pure distilled water the angle of capillarity α or the angle at which the surface of the liquid meets the glass, is zero and so $\cos \alpha = 1$.

It is important that the capillary tube be quite clean. The cleaning should be performed with chromic acid and distilled water. The height of the water in the tube can be measured in two ways. One method is to place a scale etched on mirror glass behind the tube. The mean level of the meniscus-shaped surface of the liquid in the tube and the ordinary plane surface of the liquid in the vessel should be read. A preferable method is to measure the distance between the two surfaces with a cathetometer (see p. 19).

To make certain that the inner surface of the tube is wet by the water and that the angle of capillarity is zero, the tube should be thrust deeper into the liquid and then withdrawn before the levels of the surfaces are read. This should be repeated and the height read several times, different parts of the scale being used, but the part of the tube in which the liquid rises remaining the same. If the motion of the liquid in the tube is sluggish or uncertain, the tube should be more carefully cleaned. Finally, the point to which the liquid rises in the tube should be marked on the tube by a sharp file.

The tube should then be carefully broken at the point marked and its diameter should be carefully measured by means of a micrometer microscope (see p. 15). If the section of the bore is not circular, the greatest and least diameters should be carefully measured and the mean taken, but if they differ very much the result will not be satisfactory.

The whole should be repeated with as many tubes of different sizes as time will permit. The temperature at which the work is performed should be stated.

If time permit, determine also the surface tension of an assigned solution.

Make an estimate of the possible error for the results obtained by one of the tubes.

(Apparatus for determining the surface tension at different temperatures is described in Findlay's Practical Physical Chemistry, p. 78; and Ewell's Physical Chemistry, p. 117.)

Questions.

1. Are the errors of measurement sufficient to explain the differences between results with different tubes?
2. What other sources of error may there be?
3. How could the surface tension of mercury be obtained in an analogous way?
4. How high would this liquid rise in a tube 0.1 mm. in diameter?

HEAT.

25. Radiation Correction in Calorimetry.

Watson's Practical Physics, §82; *Ostwald's Phys. Chem. Meas.*, p. 124–127; *Poynting and Thomson, Heat*, Chap. XVI.

A body which is above the temperature of surrounding bodies falls in temperature at a rate that is proportional to the excess of its temperature above that of its surroundings. This is Newton's Law of Cooling. If the mean excess of the body's temperature in any time be known and also its rate of loss of temperature at some particular ex-

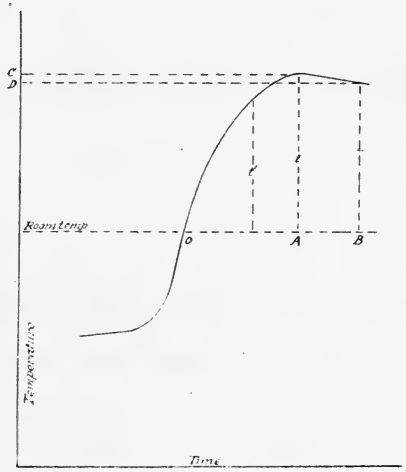

FIG. 16.

cess, its mean rate of loss of temperature is readily deduced, and this multiplied by the time for which the mean is taken will give the whole loss of temperature.

Consider the case of the heating of a vessel containing water by the passage of steam into the water. If a curve (Fig. 16—from o upwards) showing the rise of temperature of

the water be drawn and the same continued after the water has reached its highest temperature, the latter, or straight line part of the curve, will give the rate of loss of temperature at the highest temperature attained. Let us denote this rate by r (degrees per minute). If the excess of temperature when the temperature is highest is t and if the mean excess during the whole time of rise of temperature is t' then the mean rate of cooling was by Newton's Law rt'/t and this multiplied by the whole time of rise of temperature, T (minutes), gives the whole loss of temperature. Hence the final (highest) temperature must be corrected by addition of rTt'/t. If the curve of rise of temperature is a straight line, t' is half of t and the correction is $rT/2$. When the curve is not approximately a straight line the whole time T must be divided into a number of intervals (each perhaps of 30 sec.) and t' must be obtained by averaging the excesses in these intervals.

When the calorimeter is cooled below the temperature of the room (e. g., by adding ice to the water) the calorimeter gains temperature by radiation from the surroundings; but the above method will still apply except that we shall have to do with rates of warming instead of rates of cooling and the correction of the final temperature will be subtractive.

If the calorimeter starts below the temperature of the room and is heated above it, the correction must be made in two parts as above. In this case we must find the initial rate of warming (before the hot body is placed in the calorimeter) and also the final rate of cooling (after the highest temperature was attained). The correction will also be in two parts when the calorimeter starts above the room temperature and ends below it.

If the main rise of temperature is closely represented by a straight line, it is easily shown* that the correction amounts to the algebraic average of the initial and final rates multiplied by the whole time that the calorimeter is heating or

* Ewell's Physical Chemistry, p. 84.

cooling. In fact, if the water is T_1 minutes below the temperature of the room and T_2 minutes above the room temperature, the radiation correction is $(T_2r_2 - T_1r_1)/2$ and this differs from $(T_1 + T_2)(r_1 + r_2)/2$ by $(T_1r_2 - T_2r_1)/2$, which is zero, since under these circumstances the rate is proportional to the time that the water is above or below the room temperature. This fact is particularly useful in cases where the surrounding temperature is indefinite (Exp. XXVII, for example).

In every calorimetry experiment where the temperature changes, this radiation correction must be applied, and therefore the initial and final rates of change of temperature must be determined. The rate may usually be found with sufficient accuracy by reading the temperature every minute for five minutes. In very accurate work, more careful methods must be applied.

26. The Beckmann Thermometer.

Watson's Practical Physics, §102; *Findlay, Practical Physical Chemistry*, pp. 114–117; *Ostwald, Phys. Chem. Meas.*, pp. 119–120.

The *Beckmann thermometer* is used for determining changes in temperature. The bulb is large and the stem is small so that a small change of temperature is shown by a large change in reading. The amount of mercury may be varied, and the temperature corresponding to a particular reading will vary with the amount of mercury in the bulb and stem. There is a reservoir at the end of the stem into which surplus mercury may be driven by warming the bulb. A *gentle* jar will detach the mercury in this reservoir when sufficient has been expelled. If one desires to study high temperature changes, the bulb is warmed until the thread of mercury extends to the reservoir, when the mercury in the reservoir is joined to it. The bulb is then allowed to cool until sufficient mercury has been drawn over, when the thread is detached from the mercury in the reservoir by a gentle jar. Several trials are often necessary before the

proper amount of mercury is secured. In an improved type of Beckmann thermometer, two reservoirs are provided, and the first has a scale which tells the amount of mercury required in that reservoir for different ranges of temperature.

Beckmann thermometers are delicate and expensive and must be handled with the greatest care.

XIII. THERMOMETER TESTING.

Watson's Practical Physics, §§59–69; *Edser, Heat*, pp. 23–36; *Textbook of Physics (Duff)*, pp. 189, 190; *Watson's Physics*, §§177–182; *Ames' General Physics*, pp. 220–224; *Crew's General Physics*, §§249–252.

The readings of a thermometer gradually change for a long time after the thermometer has been filled. The cause of this is the gradual recovery of the bulb from the effect of the very great heating to which the glass was subjected when the thermometer was made. The shrinkage is rapid at first and slower afterward, but may continue for years. Hence the necessity for re-determining, from time to time, the so-called "fixed points" of a thermometer, namely, the reading in melting ice, and that in steam at standard pressure. When the thermometer is first graduated it is usually done by determining the fixed points and dividing the distance between them into 100 equal parts laid off on the stem. This assumes that the bore is uniform or that, by calibration of the bore, the variations of the bore are determined and allowed for in a table of corrections to be applied to the readings of the thermometer in order to obtain the true temperature. Usually the variations of the bore are too small to have any appreciable effect except in cases where extreme accuracy is aimed at. Nevertheless, every thermometer needs to be carefully examined in this regard. Let us suppose that on the scale laid off on the stem the true readings in ice and steam have been obtained and for the moment let us suppose that the bore is quite uniform. To see how to make corrections for other points on the scale we must consider the elementary definition of temperature.

Temperature on the mercury scale is defined by the expansion of mercury (relatively to glass). Let v_{100} be

the volume of a mass of mercury at the temperature of steam under a pressure of 76 cm. and let v_0 be its volume at the temperature of melting ice. The degree is defined as the rise of temperature that would produce an expansion of $(v_{100}-v_0)/100$, and $T°$ above zero is, therefore, the rise of temperature that will produce an expansion of $T(v_{100}-v_0)/100$. Hence if at $T°$ the volume of the mercury be v,

$$T\frac{v_{100}-v_0}{100}=v-v_0$$

$$\therefore\ T=\frac{v-v_0}{v_{100}-v_0}100.$$

This definition depends only on the expansion of mercury and the expansion of the particular glass used and is otherwise independent of the size and shape of the thermometer. Now regard the thermometer tube under test as simply a graduated cylinder of constant cross section containing mercury. Let the height of the mercury as read on the scale when the thermometer is in melting ice be a; when it is a steam at 76 cm. let it be b, and when it is at the temperature T, let it be t. Then

$$\frac{v-v_0}{v_{100}-v_0}=\frac{t-a}{b-a}.$$

Hence

$$T=(t-a)\frac{100}{b-a},$$

where T is the true temperature when the reading of the thermometer is t. By this equation values of T for values of t for every five degrees should be calculated. Having thus drawn up a table of true temperatures we subtract the scale-reading from the true temperature and thus get a correction (positive or negative), which *added to* the scale-reading gives the true temperature.

This is on the assumption that the bore is sensibly uniform. The only quite satisfactory method of testing this

is to calibrate the bore by measuring the length of a short thread of mercury at different positions in the tube. This process requires considerable time and the following will usually suffice: Two thermometers for which tables of true readings have been drawn up as above, are compared at regular intervals (say every five degrees) between zero and 100° by being used simultaneously to measure the temperature of a body. If, after corrections, the readings of the thermometers are not sensibly different, this shows that the bores of both must be practically uniform. If they do differ appreciably, then the bore of one or both must be variable. If they be compared with a third thermometer, the one with the variable bore will be detected and it must be then calibrated.

Testing Zero-point.—A calorimeter consisting of a small copper vessel inside of a larger is suitable for holding the ice. Both vessels should be washed in ordinary tap water. The space between the two vessels should be filled with cracked ice, and the inner vessel filled with cracked ice and then distilled water poured in until the vessel is filled to the brim. The thermometer having been washed clean, is inserted in the inner vessel, just sufficient of the stem being exposed to admit of the zero being observed. When the reading has fallen to 1° the reading should be observed every minute until it is stationary for five minutes. This stationary temperature, read to 0.1 degree, is the true zero point, or a in the above equation.

Sources of Error.

(1) Impurity in the ice or water.

(2) The presence of water above 0° near the bulb of the thermometer.

Testing Boiling-point.—The form of boiler used for this test consists of a vessel for boiling water surmounted by a tube up which the steam passes, this tube being enclosed in another down which the steam passes to an exit tube and a pressure gauge (see Fig. 17). Half fill the lower part of the vessel with water. Push the thermometer to be tested

through a cork in the top until the boiling-point is only a degree or two above the cork, but take care that the bulb of the thermometer does not reach down to the water. Apply heat, adjusting it carefully as boiling begins, so that the pressure inside, as indicated by the pressure gauge, shall not materially exceed atmospheric pressure. Some excess is, of course necessary, if there is to be a free flow of steam. What excess is permissible may be deduced from the consideration that a rise of pressure of 1 cm. (of mercury column) corresponds to a rise of boiling-point of 0.373° (see Table XIII). If water is used in the pressure gauge, a pressure of 1 cm. of water column would correspond to only 0.03° rise of steam temperature. If the thermometer be graduated to degrees only, an error of 0.03° in finding the boiling-point is negligible.

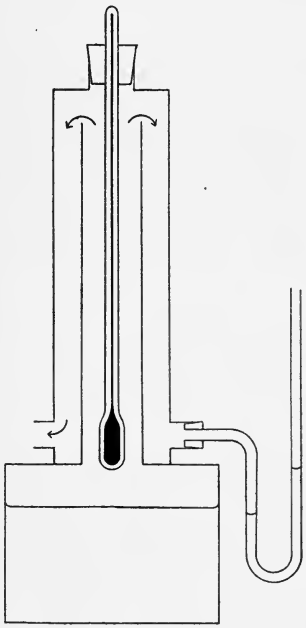

FIG. 17.

Read the barometer and reduce the height to zero degrees (p. 21).

To the boiling-point thus found a correction must be applied, for the difference between the atmospheric pressure at the time and that of a standard atmosphere (76 cm. of mercury). Find from Table XIV the true temperature of the steam at this pressure, and the difference between the boiling-point observed on the thermometer and this temperature. Since this temperature is always within a few degrees of 100°, the thermometer will have practically the same error at 100°. Therefore b in the above equation may be taken as 100 plus or minus the difference between the observed boiling-point and the true boiling temperature.

Comparison of Two Thermometers.—The most satisfactory method is to immerse the thermometers in steam above water boiling under a pressure that can be regulated. A simple means that is sufficient if the thermometers are of the same length and graduated to degrees only, is to use the thermometers simultaneously to find the temperature of a block of good conducting material (copper or brass) immersed in a vessel of water the temperature of which can be gradnally raised by a burner. The thermometers should be thrust in holes close together in the block and before each reading the burner should be removed and the water well stirred for a minute so that the temperature of the block shall become uniform.

Questions.

1. Which should be determined first, boiling-point or freezing-point, and why?
2. How much error might there be in determining the boiling-point if only the bulb were immersed in the steam?
3. Why is there no need to take account of barometric pressure in finding the zero-point?

XIV. TEMPERATURE COEFFICIENT OF EXPANSION.

Edser, Heat, pp. 39–61; *Text-book of Physics* (*Duff*), pp. 194–197; *Watson's Physics*, §§184, 185; *Ames' General Physics*, pp. 229–233; *Crew's General Physics*, §§263–265.

For measuring the thermal expansion of a body, choice may usually be made from a variety of methods. The particular method chosen will depend on the form of the specimen. The expansion of a metal rod may be measured by means of a spherometer or by means of two reading microscopes focused on definite marks near the ends of the specimen. The expansion of a wire is best measured by an optical lever method. The expansion of a solid of irregular form can be found by a hydrostatic method, namely, by weighing it in a liquid at different temperatures, it being supposed that the density of the liquid at different temperatures is known.

(A) *Expansion of a Metal Rod.*—The rod is supported at the lower end on a firm point and is heated by being enclosed in a tube through which steam is passed from a boiler. A spherometer (see p. 16) is so supported that the end of the screw can be brought down on the flat end of the rod. The spherometer is supported in the hole, slot, and plane method, so that its position is definite and not liable to be disturbed by thermal expansion of the supporting surface.

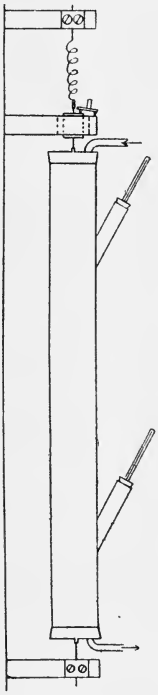

FIG. 18.

The rod is first measured by means of an ordinary meter scale divided to mms. It is then accurately placed in position in the heating tube, the end of the rod projecting through corks. Through the cork at the upper end should also pass a glass tube for the entry of the steam, while a similar tube at the lower end serves to drain off the water.

At least six readings of the spherometer scales should be made at the room temperature. Then pass steam into the jacket about the rod. Every few minutes read the temperature of the interior as given by two thermometers at different heights and read the spherometer. When the temperature has become constant, make at least six readings of the spherometer and several readings of the thermometer. Always estimate tenths of the smallest division. From the difference in spherometer readings, the length, and the change in temperature, calculate the coefficient of expansion.

(B) *Expansion of a Wire.*—For this an optical lever method is most suitable. A mechanical lever or system of levers is sometimes employed for magnifying small motions. A ray of light reflected from a mirror that is tilted by the expansion serves the purpose of a long index arm much better, inasmuch as it has no weight itself and may

be taken as long as we wish. The wire is hung vertically, the lower end being solidly clamped, and the upper end carrying a sleeve on which rests one leg of a small three-legged bench, on which a mirror is mounted. The other two legs rest on a fixed bracket. The wire is enclosed by a tube through which a current of steam is passed from a boiler and into which two thermometers are thrust to read the temperature. A drainage tube at the lower end allows the escape of water. The wire is prolonged above the mirror and is attached to a spring by which the wire is kept stretched. The image in the mirror of a vertical scale is observed by a reading telescope (see p. 25 for adjustments), and the change of reading, d, on the horizontal cross-hairs of the telescope, produced by the expansion is noted. Let the length of the wire between the clamp and the support of the bench be l, and let the length of the bench between the point of the movable leg and the line of the other legs be a. Let the distance of the scale from the mirror be L, and the change of temperatures be $(t_2 - t_1)$. Then, remembering that a ray of light reflected from a mirror turns through twice the angle that the mirror turns through, it is easily seen from the figure that the expansion is $ad/2L$ and the coefficient of expansion is

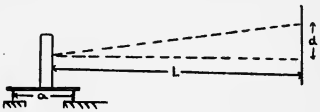

FIG. 19.

$$e = \frac{ad}{2Ll(t_2 - t_1)}$$

The most difficult quantity to determine with a high degree of precision is a. It may be measured by means of a micrometer microscope or a dividing engine (see p. 17). A simpler and more accurate method is to place the optical lever so that the movable leg is on the (vertical) screw of a micrometer caliper (p. 14), while the other legs are on a fixed support and then focus the telescope and scale on the mirror. When the screw is turned the movable leg

is raised a known amount. From this, the distance between the mirror and the scale, and the scale-readings, a is deduced.

This calibration may be avoided by placing two legs of the mirror bench upon the collar attached to the wire and resting the third leg directly upon the micrometer screw. The extension may also be measured by Searle's combination of level and micrometer screw (see end of Exp. IX).

XV. COEFFICIENT OF APPARENT EXPANSION OF A LIQUID.

Edser, Heat, pp. 64–71; *Text-book of Physics (Duff), pp.* 198–201; *Watson's Physics*, pp. 211–213; *Ames' General Physics*, pp. 233–235; *Crew's General Physics*, §§266, 267.

The object of this experiment is to determine the *coefficient of apparent expansion* of some salt solution with reference to glass. A vessel holds M grams of liquid at $t°$ and m grams at a higher temperature, $t'°$. Let V be the volume of the vessel at the lower temperature. Since we are considering the apparent expansion, i. e., the expansion with reference to the vessel, we may consider V to be also the volume of the vessel at the higher temperature. The volume of 1 gram at $t°$ is therefore V/M and at $t'°$, V/m. The increase in volume is

$$\frac{V}{m}-\frac{V}{M}=V\left(\frac{M-m}{Mm}\right).$$

The coefficient of apparent expansion, e, is this apparent expansion divided by the original volume V/M and the range of temperature $(t'-t)$ or .

$$e=\frac{M-m}{m(t'-t)}.$$

A glass bulb with a re-curved capillary stem is used. *To fill the bulb with a liquid*, warm it with the hand or by *playing* a flame about some distance beneath it. Remove it from the source of heat and plunge the end of the stem into the liquid. As the air in the bulb cools liquid will be drawn in.

To expel liquid, warm the bulb gently, keeping it so turned that the stem is filled with the liquid; when the liquid ceases to come out, invert it so that the stem is highest, and allow it to partially cool. Repeat until all the liquid is expelled.

Clean the bulb by drawing in a little distilled water, or, if the interior be foul, first use chromic acid. Finally rinse the interior with alcohol. Remove the alcohol and dry the interior, if necessary playing a flame about some distance beneath.

To determine the density of the (cold) salt solution, thoroughly cleanse a tall measuring glass and a suitable hydrometer (variable immersion). Pour enough of the salt solution into the measuring glass to float the hydrometer, read the density, and pour the solution back into the bottle.

Weigh the bulb very carefully on a sensitive balance (see pp. 21–25). Support the bulb in a clamp stand, clasping the stem between half corks. Fill a small beaker with the salt solution and support it so that the end of the stem dips into the solution. Warm the bulb, playing a Bunsen flame beneath. *Never allow the flame for an instant to remain stationary beneath the bulb, and until the bulb contains considerable warm liquid, do not allow the flame to touch the bulb, and then only where there is liquid.* Alternately warm the bulb and allow it to cool a little until the bulb is filled. When it is partly full it may be best to *gently* boil the liquid in the bulb. When the bulb is almost full the liquid can be made to expand to fill the entire stem. Then allow it to cool completely while it draws over liquid from the beaker.

When the bulb is cooled to the temperature of the room, support it in a copper vessel in which water is kept at a constant temperature, a *few degrees warmer than the room.* When the temperature has been kept constant for five minutes (by the addition of small amounts of hot or cold water, if necessary) and has been frequently stirred, read the temperature (as always estimating tenths). Remove any liquid adhering to the end of the stem, remove the bulb from the bath, dry the exterior, and weigh. Handle the

bulb carefully with a cloth about it so that no liquid may be expelled. Weigh a small, clean, dry beaker. Support the bulb again in the copper bath with the beaker beneath the end of the stem, to catch any liquid expelled. Heat the water in the bath to boiling. When the temperature has been constant for five minutes, read the temperature, catch on the side of the small beaker any liquid adhering to the end of the stem, remove the bulb from the bath, dry the exterior, and weigh. Weigh the small beaker with the liquid contained. Carefully remove the liquid from the bulb and stem as described above.

The difference between the two weights of the bulb when filled with liquid gives the weight $M-m$ of liquid expelled. The difference between the weight of the flask dry and after being in the second bath gives the final weight of liquid in the bulb. The expelled liquid is saved simply as a check and is not used at all if the above difference be slightly greater.

A specific-gravity bottle may be substituted for the bulb, but is not as satisfactory.

Questions.

1. Why is double weighing unnecessary?
2. Why is $M-m$ determined more accurately from the difference of the two weighings than from the weight of the liquid expelled.
3. How might the coefficient of expansion of a solid, attainable only in the form of small lumps, be found by an extension of this method?
4. How might the absolute expansion of a liquid be found by the above apparatus?

XVI. COEFFICIENT OF INCREASE OF PRESSURE OF AIR.

Edser, Heat, pp. 106–111; *Poynting and Thomson, Heat*, pp. 45–49; *Text-book of Physics (Duff)*, pp. 188, 203–205; *Watson's Practical Physics*, §§78, 79; *Watson's Physics*, §§195–198; *Ames' General Physics*, p. 240; *Crew's General Physics*, §269.

If the volume of a mass of gas remains constant while its temperature is raised, its pressure increases according to the law

$$P = P_0(1 + a\,t),$$

in which P_0 is the pressure at 0° C., P the pressure at the temperature t, and a is a constant called *the coefficient of increase of pressure.* If the pressure were kept constant and the volume allowed to increase, the law of increase of volume would be similar, and it is found that the constant a is practically the same in both cases.

It is, however, difficult to keep the volume exactly constant, for the containing vessel will expand when heated (the volume of the vessel would also increase because of the increase of pressure to which it is subjected, but this may be neglected since it is extremely small). If p is the observed pressure at temperature t and P_0 the observed pressure at 0°

$$p = P_0(1 + a't),$$

where a' *is the coefficient of apparent increase of pressure* (see Exp. XV).

To correct for the expansion of the vessel, we must suppose the final volume of the gas compressed in the proportion in which the capacity of the vessel expanded. The law of expansion of the vessel is

$$v = v_0\ (1 + b\ t),$$

where b is the coefficient of cubical expansion of the vessel. To get the pressure P that would keep the volume of the gas absolutely constant, we must multiply p by $(1 + b\ t)$,

$$\therefore\ P = P_0(1 + a't)(1 + bt)$$
$$= P_0\{1 + (a' + b)t\}.$$

And so the true coefficient of increase of pressure a is obtained from the apparent coefficient of increase of pressure a', by adding the coefficient of cubical expansion of the vessel, or,

$$a = a' + b.$$

The air (or gas) is enclosed in a bulb to which is connected a mercury manometer. The pressure indicated by the manometer is obtained from readings of the mercury levels on a scale between the two columns, or, preferably, with a cathetometer (p. 19).

If the true increase of pressure of dry air is desired the air must first be carefully dried. To fill the bulb with dry air it may be connected through a drying tube (containing chloride of calcium) with an air-pump and the bulb several times exhausted and refilled with air sucked through the drying tube. (If the bulb be already filled with dry air the process will be unnecessary.)

The bulb is then connected to the manometer. The bulb is first immersed in a bath of ice and water as nearly as possible at 0°, and the movable column of the manometer is adjusted until the mercury in the other column is at a definite point, as high as possible without entering the contraction where connection is made with the bulb. The temperature and pressure are read as carefully as possible, at least six times, when both have become quite steady, the manometer being readjusted before each reading.

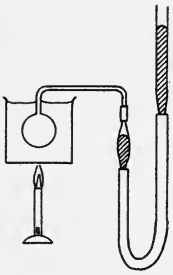

FIG. 20.

The bath of ice and water is now replaced by one of water at about 10° and the movable column is readjusted until the mercury in the stationary column is at the former point, that the volume of the gas may remain constant. The temperature and pressure are read when they have become steady. The water is then heated to about 20° and the observations are repeated. Readings are thus made at intervals of about 10° until the water boils.

The pressure and temperature when the water is boiling should be read at least six times, the mercury level in the stationary column being adjusted to the constant point before each reading. It is at the initial temperature (near 0°) and the final temperature (near 100°) that the most reliable observations are obtained, and it is upon these that the most reliable estimate of the coefficient of expansion is founded. (The readings at intermediate temperatures are made in order to test the law of expansion.) If the two arms of the manometer are of different radii, there will be a con-

stant difference of level due to capillarity. This should be read when the bulb is disconnected and allowance should be made for it at other times. Read the barometer (p. 21) and the temperature of the barometer and of the mercury in the manometer.

Tabulate from your observations (a) the temperatures; (b) the differences in level of the mercury columns; (c) these differences reduced to zero degrees; (d) the pressures as calculated from (c) and the barometer heights (reduced to zero degrees).

The test of the law of expansion is made by plotting the curve of pressure and temperature, the former as ordinates, the latter as abscissas. This should be nearly a straight line. The averages for the first point (0°) and the last (about 100°) are to be taken as fixing the straight line. The divergence of intermediate points from the straight line, while not sufficient to invalidate the conclusion that the increase of pressure is linear, will illustrate the difficulty of keeping the temperature at intermediate points constant for a sufficient length of time for the air in the bulb to come wholly to the temperature of the water.

Calculate from these two average pressures and temperatures, the coefficient of apparent increase of pressure (a'), and, obtaining the coefficient of cubical expansion of the glass (b) from Table VIII, find the true coefficient of increase of pressure (a). (Remember that the coefficient of cubical expansion is three times the coefficient of linear expansion.)

If time permit, increase the range of temperature by observations below 0° in a freezing mixture and above 100° in heated oil.

Questions.

1. Why must (a) the air be dry? (b) a capillary connect the bulb and the manometer?
2. What would be the percentage error if the expansion of the bulb was neglected?

XVII. PRESSURE OF SATURATED WATER VAPOR.

Poynting and Thomson, Heat, Chap. X; *Edser, Heat*, pp. 220–228; *Text-book of Physics (Duff)*, pp. 226–230; *Watson's Physics*, §§216–218; *Ames' General Physics*, pp. 264–269; *Crew's General Physics*, §§279–281.

The object of this experiment is to find the *pressure of saturated water vapor* at different temperatures. By pressure of saturated water vapor at a given temperature, or, as it is often called, maximum pressure of water vapor, or, equilibrium pressure, is denoted the pressure of the vapor above water in a closed vessel at the given temperature after a steady state has been reached. A liquid continues to give off vapor *from the surface*, or, "*evaporate*," as long as the pressure of the vapor above the liquid is less than the saturated vapor pressure, independent of the total atmospheric pressure above the liquid. After the pressure of the vapor reaches the saturated vapor pressure for that temperature, the total quantity of vapor in the atmosphere above the liquid remains constant, since for any vapor given off from the surface an equal quantity is condensed.

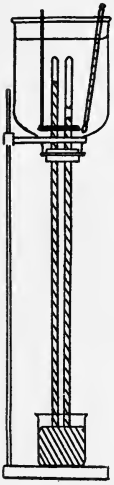

FIG. 21.

There are two chief methods of finding the saturated vapor pressure, the static method and the kinetic method.

(A) In the *static method* some water (or other liquid) is introduced into the space at the top of a barometric column which is surrounded by a bath, the temperature of which can be varied. The pressure of the vapor is found by measuring with a cathetometer (p. 19) the height of the mercury column and subtracting this from the barometric reading, each being reduced to zero (p. 21). By varying the temperature of the bath, the vapor pressure at various temperatures is obtained.

(B) In the *kinetic method* the *quantity measured* is the

temperature of the steam above water boiling under different *measured pressures*. When a liquid boils, bubbles of vapor are formed *throughout the interior* of the liquid. In forming these bubbles, the vapor overcomes the pressure of the atmosphere above the liquid, therefore the pressure of the vapor must equal the atmospheric pressure, and obviously the vapor in the bubbles is saturated. Hence, in measuring the atmospheric pressure above a liquid boiling at a known temperature, we find the saturated vapor pressure of the liquid at this temperature, and this is Regnault's method, which method is followed in this experiment.

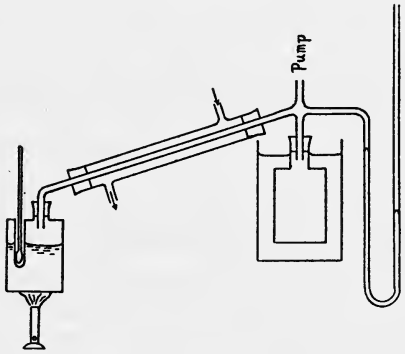

FIG. 22.

In *Regnault's apparatus* the total pressure above the surface of the liquid can be kept very constant. As the liquid is heated, the vapor is condensed in a Liebig condenser, and as the pressure of vapor distributed through several conducting vessels is the vapor pressure corresponding to the vessel at lowest temperature, the pressure exerted by the vapor cannot exceed the maximum pressure corresponding to the temperature of the tap water, and is therefore very small. As the temperature of the boiler changes, the temperature of the air in the boiler varies, but a large air reservoir surrounded by water is connected between the condenser and the manometer and air-pump or aspirator, which makes the volume of the air in the boiler small com-

pared with the total volume of air in the system, and thus the increase of pressure due to the heating of the air in the boiler is small.

The *boiler* should be about two-thirds full of water. Fill with water the small tube running down into the boiler (which tube is closed at the bottom), and insert in this tube through a cork one of the thermometers tested by the observer. Draw out any water which may be in the *air reservoir* by means of the stopper underneath. Fill the surrounding vessel with water. (Rubber stoppers should be lubricated with rubber grease (note p. 32) before insertion.) Exhaust the air from the system to the highest vacuum attainable by means of a Geryk pump or aspirator. Close all the cocks through which connection is made to the aspirator and let the system stand a few minutes to see if there is any leakage. If not, start a gentle stream of water through the condenser, and place a Bunsen flame under the boiler. *Read the barometer* and its temperature (see p. 21).

When the temperature as registered by the thermometer in the boiler becomes very steady, read it, and at once record the two extremities of the mercury column of the manometer. Let in a little air by first opening and then closing a cock near the air-pump, and then opening and closing a cock nearer the apparatus. Increase the pressure at first by about 15 mm., gradually increasing the steps and when near atmospheric pressure change the pressure by about 12 cm. The reason for the difference in pressure in the steps is that it is better to have the steps represent about equal changes of temperature, for instance, about 5°.

From the corrected barometer reading and the differences in height of the mercury columns, calculate the pressures. Tabulate pressures and temperatures and also plot them, making temperatures abscissas and pressures ordinates.

Ramsay and Young's method for measuring the vapor pressure of a small quantity of liquid is described in *Watson's Practical Physics*, §94.

Questions.

1. State precisely what two quantities you have observed in the second method and what relation they bear to the pressure and temperature of saturated vapor.
2. What condition determines whether a liquid will boil or evaporate at a given temperature?
3. What was the actual vapor pressure above the boiling liquid? (Table XIII.)
4. What determines (a) the lowest temperature, (b) the highest temperature for which this apparatus is applicable?

XVIII. HYGROMETRY.

Poynting and Thomson, Heat, pp. 209–215; *Davis, Elementary Meteorology*, Chap. VIII; *Robson, Heat*, §§73–75; *Watson's Practical Physics*, §§95–97; *Text-book of Physics* (*Duff*), pp. 239–242; *Watson's Physics*, §§220, 221; *Ames' General Physics*, pp. 265–268.

Three methods will be used for studying the hygrometric state of the atmosphere. The first method (A) determines the dew-point, the second (B) determines, indirectly, the actual vapor pressure, and the third (C) determines the relative humidity.

(A) *Regnault's Hygrometer.*—A thin silvered-glass test-tube is half-filled with ether. The test-tube is tightly closed by a cork through which passes a sensitive thermometer which gives the temperature of the ether. Two glass tubes also pass through the cork, one extending to the bottom, the other ending below the cork. An aspirator gently draws air from the shorter tube. The ether is evaporated by the air bubbles and the entire vessel cools. The silvered surface and the thermometer are watched through a telescope and the temperature is read the moment moisture appears on the metal. The air current is stopped and the temperature of disappearance of the moisture is observed. This is repeated several times and the mean is taken as the *dew-point.* The detection of moisture is facilitated by observing at the same time a similar piece of silvered glass which covers a part of the test-tube, but which is insulated from it. The temperature of the air

should also be carefully determined, preferably with a thermometer in a similar apparatus where there is no evaporation.

An arrangement of two small mirrors at right angles so placed as to reflect light from the two tubes into the telescope will facilitate the comparison.

(B) *Wet and Dry Bulb Hygrometer.*—Two thermometers are mounted a few inches apart. About the bulb of one is wrapped muslin cloth to which is attached a muslin wick dipping in water. The other is bare. The temperatures of both are read when they have become steady. The temperature of the first thermometer will be lower than that of the bare thermometer, on account of the evaporation of the water. From the difference of temperature of the two thermometers and the temperature of the bare thermometer the actual vapor pressure may be determined with the aid of empirical tables (see Table XV). For more accurate apparatus, see references.

(C) *Chemical Hygrometer.*—Fill three ordinary balance drying vessels with pumice. Saturate two with strong sulphuric acid and the third with distilled water. Weigh very carefully the two which have the acid and then connect them to an aspirator, with the water absorption vessel between them. After a gentle stream of air has passed through for a considerable time, disconnect and weigh the sulphuric acid vessels. The ratio of the gains in weight will obviously be the relative humidity. Observe also the temperature of the air.

If not directly determined, calculate from your observation, by each of the three methods, the dew-point, the actual vapor pressure, the relative humidity, and the amount of moisture in the atmosphere per cubic meter. Tabulate your results. Table XIII gives the vapor pressures of water at different temperatures.

XIX. SPECIFIC HEAT BY THE METHOD OF MIXTURE.

Edser, Heat, pp. 122–136; *Text-book of Physics (Duff)*, pp. 208–211; *Watson's Physics*, §§200–201; *Watson's Practical Physics*, §§82–84; *Ames' General Physics*, pp. 250–252; *Crew's General Physics*, §252.

The *specfic heat* of a substance is the number of calories required to raise the temperature of one gram of the substance one degree centigrade, or the number of calories given up by one gram in cooling one degree centigrade. In the method of mixture a known mass (M) of the substance, heated to a known temperature (T), is immersed in a known mass of liquid (m) of known specific heat (for water $=1$), at a known temperature (t_0), and the unknown mean specific heat (x) of the substance is deduced from these data and the temperature (t) to which the mixture rises. Water is the liquid employed unless there would be a chemical reaction on immersion.

The liquid must be contained in a vessel which is also heated by the immersion of the hot body. The heating of the vessel is equivalent to the heating of a certain additional quantity of water. This equivalent quantity of water (e) is called the water equivalent of the vessel. It is practically equal to the mass of the vessel (m_1) multiplied by the specific heat (s) of the material of the vessel. Theoretically it may be obtained by noting the temperature of the vessel and pouring into it a known mass of water at a known temperature and noting the final temperature. This is an inverted form of the method of mixture applied to finding the specific heat of the vessel. But as we shall presently see, it is the method of mixture applied under very unfavorable conditions and will not usually give a very satisfactory result. Another method will be recommended below.

The equation for finding the specific heat is obtained by equating the heat given up by the hot body to that taken up by the water and containing vessel. Hence

$$Mx(T-t)=(m+e)(t-t_0). \qquad (1)$$

Sources of Error.

(1) Loss of heat while the hot body is being transferred to the water.

(2) Loss of heat by radiation, conduction, or evaporation while the mixture is assuming a uniform temperature.

(3) Errors in ascertaining the true temperature including errors in the thermometers.

Choice of Best Conditions.—As the accuracy of this determination depends largely on the selection of suitable conditions, we shall consider how these may be chosen so that unavoidable errors in the separate measurements may affect the result as little as possible.

By taking the logarithms of both sides of (1) and differentiating partially, we obtain (see pp. 7, 8.)

$$\text{(2)}\quad \left[\frac{\delta x}{x}\right]M = -\frac{\delta M}{M}, \qquad \text{(3)}\quad \left[\frac{\delta x}{x}\right]m = \frac{\delta m}{m+e}, \qquad \text{(4)}\quad \left[\frac{\delta x}{x}\right]e = \frac{\delta e}{m+e},$$

$$\text{(5)}\quad \left[\frac{\delta x}{x}\right]T = -\frac{\delta T}{T-t}, \qquad \text{(6)}\quad \left[\frac{\delta x}{x}\right]t_0 = -\frac{\delta t_0}{t-t_0}, \qquad \text{(7)}\quad \left[\frac{\delta x}{x}\right]t = \frac{(T-t_0)\delta t}{(T-t)(t-t_0)}.$$

The left-hand side of (2) stands for "the ratio that the possible error (δx) in x, due to the possible error (δM) in M, bears to x," and so for the other equations.

M and m can be measured with great precision; hence (2) and (3) are negligible. From (4) it is seen that the water equivalent of the calorimeter must be found with some care. From (5) and (6) it is seen that the ranges $T-t$ and $t-t_0$ should be as great as possible (see, however, "sources of error" above). This is also consistent with the indications of (7), for although $(T-t_0)$ enters the numerator, the product of $T-t$ and $t-t_0$ is in the denominator. Moreover, it is seen from (5), (6), and (7) that if equal errors are made in observing T, t, and t_0, the effect of the error in t may equal the sum of the effects of the errors in T and t_0. Hence the necessity of determining t with special care. But, allowing an unavoidable error in

t, how can its effect be made as small as possible by properly choosing the quantities, M, m, t_0, T? Let us suppose $T-t_0$ is taken as great as possible under the circumstances. How can $(T-t)\times(t-t_0)$ be made as great as possible? The sum of $T-t$ and $t-t_0$ is $T-t_0$, a fixed quantity. Hence their product is, by algebra, a maximum when they are equal or t is midway between T and t_0. But it is seen from (1) that this also requires Mx and $m+e$ to be equal. Hence we see that *for the best results, T and t_0 should be as far apart as possible and the heat capacity of the specimens should be as nearly as possible equal to the heat capacity of the water and the vessel that contains it.*

The logical procedure, then, would be to roughly determine x by the method of mixture, using any convenient values of M and m, and with this rough value for x, calculate what ratio of M to m would best satisfy the above condition. Moreover, it is seen from (4) above that m should be as large as is consistent with other conditions. Then we should proceed to arrange a new experiment to be performed under the more favorable conditions for precision.

We now see why it is not easy to determine the water equivalent of the vessel directly. Its heat capacity is small compared with that of the water that would fill it, and so the change of the temperature of the water would be small and difficult to determine accurately. If a much smaller quantity of water were used, a large part of the surface of the vessel would be left uncovered, and its temperature could not be determined. Hence it is better to determine the specific heat of the material of the vessel, using the ordinary method of mixture and a mass of the same material as the vessel. Then multiplying the mass of the vessel by its specific heat, we have its water equivalent.

It is desirable that the *body* should have such a form that it and the water in which it is immersed should rapidly come to a common temperature. Filings, shot, thin strips, wire or small pieces would best satisfy this condition. Larger solid masses are more rapidly (and therefore with

less loss of temperature) transferred from the heater to the water, and to give the water ready access to them they may be perforated with holes, through which, by moving the mass up and down in the water, the water may be made to circulate. The following directions apply primarily to this latter form, but may be readily adapted to the other forms.

Two forms of heater will be here described. (1) *The steam heater.* A copper tube large enough to admit the specimen is enclosed, except at the ends, by an outer copper vessel which is to act as a steam jacket to the inner vessel. Steam from a simple form of boiler is admitted to the jacket through a tube near the top of the jacket and escapes through an outlet near the bottom. If the body to be heated is a solid mass, it is suspended in the heater by a long string that passes through a cork that closes the upper end of the heater. (If the specimen is in the form of shot or clippings they are placed in a dipper that fits into the heater.) A thermometer passed through the cork or cover of the dipper is pushed down until it comes into contact with the body tested. The lower end of the heater is also closed with a cork.

Such a steam heater ultimately brings the specimen to a very steady temperature, but it has the disadvantage of heating very gradually. If the boiler which supplies steam to the jacket has a closed tube extending from the top into the interior of the boiler, of slightly larger diameter than the specimen or dipper, either of the latter may be placed therein and rapidly heated to about the steam temperature, when they are transferred to the steam heater.

(2) *The Electric Heater.*—A metallic tube is heated by a strong current of electricity passing through a coil of wire of high resistance that surrounds the tube. The current can be varied by changing a variable resistance in circuit with the heating coil. With an alternating current the resistance may be an inductive resistance or choking coil consisting of wire surrounding a soft-iron wire core. A low resistance allowing a high current is used until the tem-

perature rises to the desired point (perhaps near 100°) and then the current is reduced to the strength that will keep the temperature constant, as indicated by a thermometer hung in the heater. The body is introduced into the heater exactly as in the case of the steam heater. The proper method of varying the resistance can only be learned by some practice.

The two *thermometers* used should be those for which tables of corrections have been obtained earlier.

The *calorimeter* may be prepared while the specimen is being heated. It consists of a smaller copper or aluminum can highly polished on the outside and enclosed in a larger one brightly polished on the inside, but well insulated from it by corks or cotton-wool. A wooden cover fits over both vessels and has holes for thermometer and stirrer and an opening giving access to the interior vessel. A convenient form has a trap-door which slides open in two halves, exposing the entire inner vessel. A screen with sliding or, preferably, double swinging doors, should separate the calorimeter from the boiler, heater, etc.

The "*water equivalent*" of the receiving vessel means the water equivalent of the inner vessel together with the stirrer, if one be used. It is advisable that the stirrer should also be of copper or aluminum. (A stirrer is, however, not necessary when the specimen is in the form of one large block).

At certain times, in the *manipulation* of this experiment the co-operation of two persons is desirable, and, for economy of time, two determinations should be made simultaneously, two heaters, two specimens, and two calorimeters being used. One specimen should peferably be of the same material as the calorimeter, so that the water equivalent may be determined.

The body whose specific heat is to be determined is weighed to 0.1 gm. and placed in the heater along with a thermometer. The inner vessel of the calorimeter (including the stirrer) is weighed to 0.1 gm. Water near the tem-

perature of the room is poured into it until it is judged that when the hot body is immersed it will be completely covered and the water will rise to within a couple of centimeters of the top of the vessel. The vessel and water are then weighed to 0.1 gm. The inner vessel is now replaced in the outer and the cover adjusted and closed.

When the temperature of the specimen has remained constant for ten minutes, it may be assumed that the hot body is throughout at the temperature of the heater. The next steps require two persons, and as it is important that it should be carried out promptly and neatly, it should be carefully considered before being performed. One person constantly stirs the water in the calorimeter and reads the temperature, to tenths of a degree, every minute for five minutes. He then opens the cover and slides the calorimeter beneath the heater. The other observer has meanwhile made a careful final observation of the temperature of the heater and removed the lower stopper. As soon as the calorimeter is in position, he lowers the hot body, without splash, into the water. The calorimeter must then be immediately removed and the cover closed.

One observer should then keep the mixture stirred by moving the body up and down with the aid of the string and note the temperature at as short equal intervals as possible (perhaps every 15 secs.) while the other records the readings. After the highest temperature has been reached, the readings are continued every minute for at least five minutes.

Simple and obvious modifications of the above procedure are required if the specimen is in the form of shot or clippings.

After rough calculations of water equivalents and specific heats, the observers should exchange duties and repeat the whole, using masses that most nearly accord with the conditions laid down in the considerations stated above. Before calculating final results, make corrections of the temperatures, T, t, t_0, according to the tables of corrections already obtained for the thermometers used.

Plot all the temperature observations of the final determination, and correct for radiation according to the directions given on pages 63, 64. The possible error of the result should be calculated as explained on pages 7, 8.)

Questions.

1. How might the present method be adapted to find the specific heat of a liquid?
2. Considering evaporation, loss of heat when transferring the hot body, and any other sources of error that may occur to you, is your result more probably too high or too low?

XX. RATIO OF SPECIFIC HEATS OF GASES.

(Clement and Desormes' Method.)

Edser, Heat, pp. 317–325; *Text-book of Physics (Duff)*, pp. 211–212, 267, 268; *Watson's Physics*, §§259–260; *Watson's Practical Physics*, §105; *Ames' General Physics*, pp. 252–256.

The gas is compressed into a vessel until the pressure has a value which we will designate by p_1. The vessel is then opened for an instant, and the gas rushes out until the pressure inside falls to the atmospheric pressure, p_0. This expansion may be made so sudden that it is practically adiabatic and the temperature of the gas will therefore fall. After the vessel has been closed for a few minutes, the gas will have warmed to the room temperature, t, and the pressure, p_2, will be above that of the atmosphere. Consider one gram of the gas. During the adiabatic expansion, its volume changed from v_1 to v_2, according to the adiabatic equation for pressure and volume (see references)

$$\left(\frac{v_2}{v_1}\right)^{\gamma} = \frac{p_1}{p_0}$$

Since the initial and final temperatures are the same, and since the volume remains v_2 while the gas is warming and the pressure is rising from p_0 to p_2, by Boyle's Law

$$p_1 v_1 = p_2 v_2$$

$$\therefore \frac{p_1}{p_0} = \left(\frac{p_1}{p_2}\right)^{\gamma}$$

Hence γ, the ratio of specific heats, is given by the equation

$$\gamma = \frac{\log \frac{p_1}{p_0}}{\log \frac{p_1}{p_2}}$$

A large carboy is mounted in a wooden case and may be surrounded with cotton batting. The neck is closed with a rubber stopper through which passes a T-tube connected on one side with a compression pump (e. g., a bicycle pump),

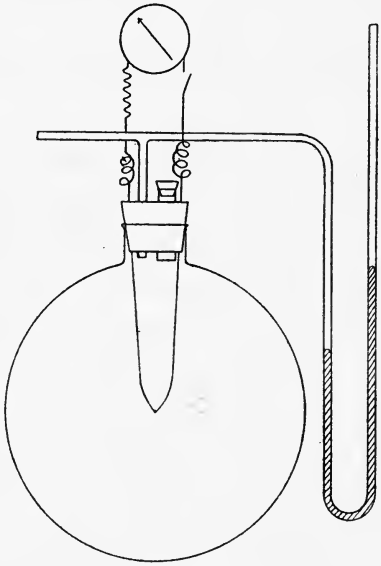

FIG. 23.

and on the other side with a manometer containing castor oil.* A large glass tube, which may be closed by a rubber stopper, also passes through this large stopper. A little sulphuric acid in the bottom of the carboy keeps the air dry. A very fine copper wire and a very fine constantin wire

* The density of castor oil is about .97, but it should properly be determined (Exp. VIII).

pass tightly through minute holes in the stopper and meet at the center of the carboy, in a minute drop of solder.

The air in the carboy is compressed until the difference in pressure is about 40 cm. of oil ($=p_1-p_0$). The tube connecting with the pump is closed, and, after waiting about 15 minutes to allow the air inside to regain its initial temperature (as shown by the pressure becoming constant), the ends of the oil column are carefully read. The carboy is now carefully surrounded with cotton batting, which may have been removed to facilitate cooling. The air inside is momentarily allowed to return to atmospheric pressure by removing, for about one second, the rubber stopper from the glass tube. After waiting until the air inside has assumed the room temperature (shown by the pressure becoming constant), the final pressure p_2 is determined. The cotton-wool had better be removed during this stage.

Connect the wires to a calibrated galvanometer (Exp. LVIII), apply the initial compression p_1, and observe the reading of the galvanometer when it has become steady. Remove the stopper as before (for not over one second), replace the stopper, and observe the galvanometer reading. The proper reading to record is the fairly steady deflection which is attained immediately after the stopper is removed. There are liable to be rapid fluctuations which should be disregarded, and of course the temperature does not long remain steady, owing to heating or cooling from the outside. Record as before the final pressure p_2. Repeat several times, starting with the same initial pressure p_1. Record the temperature of the room, t, and p_0, the height of the barometer (p. 21).

Calculate γ, the ratio of specific heats by the above equation. Calculate the change of temperature from the mean of the galvanometer deflections and the constants of the thermocouple and galvanometer.

Compare the result with T_1-T_0 where T_1 is $t+273$ and T_0 is calculated from the adiabatic equation for temperature and pressure (see references).

$$\frac{T_1}{T_0}=\left(\frac{p_1}{p_0}\right)^{\frac{r-1}{r}}$$

Unless exceedingly fine wire is employed (preferably No. 40, B. & S.), the heat capacity of the wire is relatively so great that the thermocouple will not show the full change of temperature.

Draw a curve with volumes as abscissæ, and pressures as ordinates, which will represent the changes in this experiment.

(Let specific volumes, i. e., volumes of one gram, be abscissæ. Calculate from Table VI and the laws of gases the specific volumes corresponding to the room temperature and p_0, p_1, and p_2, and draw the corresponding isothermal. Draw the horizontal line corresponding to p_0. Draw a vertical through the point corresponding to p_2 on the above isothermal. The intersection of these two straight lines will evidently be p_0, v_2).

Questions.

1. Do you see any objection to an initial exhaustion of the gas in place of the compression?
2. What are the advantages and disadvantages of a large opening? Short time of opening? Castor oil manometer?
3. How would an aneroid manometer be preferable in this experiment to a liquid manometer?

XXI. LATENT HEAT OF FUSION.

Edser, Heat, pp. 145–149; *Text-book of Physics (Duff)*, p. 225; *Watson's Physics*, §211; *Watson's Practical Physics*, §88; *Ames' General Physics*, pp. 260, 261; *Crew's General Physics*, §286.

The *latent heat of fusion* of a substance is the number of calories required to melt one gram of the substance. The most common method of measuring it is a method of mixture similar to that used in finding the specific heat of a solid. A known mass of the solid at its melting-point is placed in a known mass of the liquid at a known temperature, and the temperature of the liquid observed after the solid has completely melted. Allowance must be made for the water equivalent of the calorimeter and correction must be made

for the effect of radiation to or from the calorimeter while melting is taking place. The error due to radiation may be made small by having the liquid initially as much above the temperature of its surroundings as finally it falls below. Thus loss and gain by radiation will approximately balance. Nevertheless, since the calorimeter will probably not be the same length of time above the temperature of the surroundings as below, there will be a residual error for which correction must be made.

The calorimeter consists as usual of an inner can polished on the outside to diminish radiation, and enclosed in an outer can polished on the inside. The space between the two cans may be filled by cotton-wool to prevent air currents, and still further prevent communication of heat. The inner can is weighed, first empty and then half-filled with warm water about 15° above the room temperature. It is then placed in the outer can as described above and covered by a wooden cover having holes for thermometer and stirrer and a hinged cover giving access to the inner vessel.

The temperature is carefully noted each minute until it has fallen to about 10° above the room temperature. In the meantime, ice is broken to pieces of about a cubic centimeter in volume. These pieces are carefully dried in filter paper. A careful observation of the temperature of the water in the calorimeter having been made and the time noted, a piece of ice is dropped in without splashing and kept under water by a piece of wire gauze attached to the stirrer. The temperature is noted every half-minute as the ice melts, the water meantime being kept stirred. The rate at which ice is dropped in is regulated simply by the rate at which it can be dried and the temperature and time noted.

The process is continued until the temperature has fallen to about 10° below the room temperature. Then the addition of ice is discontinued and the temperature of the water further noted every minute for four or five minutes. Finally, the weight of the inner can and its contents is obtained in order that the mass of the ice may be deduced.

After the proper weight of ice has been ascertained, the experiment should be repeated with a single piece of approximately this weight. As there are considerable sources of error that cannot be eliminated, the whole determination should be repeated as often as time will permit.

All the temperature observations should be plotted against the time and the radiation correction determined as described on pages 63 and 64. In reporting, consider the possible error of your result so far as it depends on the possible error of your weighings and observations of temperature. State also any other sources of error that may have affected your result.

Questions.

1. What advantages are there in the use of one large lump over an equal mass of small ones?
2. Why must the water in the inner vessel be pure?
3. Is it preferable to have the air about the calorimeter moist or dry? Explain.

XXII. LATENT HEAT OF VAPORIZATION.

Ames' General Physics, p. 269; *Watson's General Physics*, §214; *Crew's Physics*, §287; *Watson, Practical Physics*, §§89–91; *Edser, Heat*, pp. 150–9; *Text-book of Physics* (*Duff*), p. 231.

The latent heat of vaporization of a substance is the number of calories required to change one gram of the substance from liquid to vapor. The usual method of measuring it is a method of mixture. A known mass of vapor, at a known temperature, is discharged into a known mass of liquid, at a known initial temperature, and the final temperature is noted. The same precautions are necessary as in finding the latent heat of fusion. The arrangement of the calorimeter is also the same. To minimize radiation the water should be initially as much below room temperature as it finally rises above, say 15°. The initial rate of warming should also be obtained, and also the final rate of cooling.

Several different forms of boiler have been devised for the purposes of this determination. Two will be briefly described.

In *Berthelot's boiler* the delivery tube passes out through the bottom of the boiler, which is heated by a ring burner that surrounds the tube. Thus the tube is so far as possible jacketed by the boiling water. The usual form of this boiler is somewhat fragile, but a good substitute may be made from a round-bottomed boiling flask the neck of which has been shortened (Fig. 24).

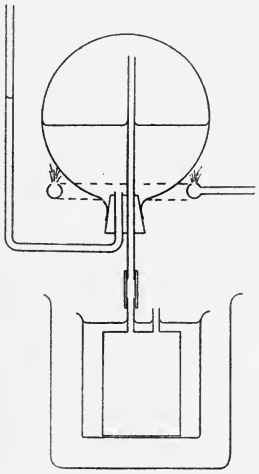

FIG. 24.

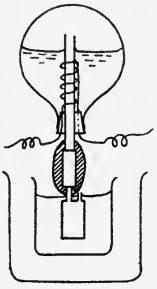

FIG. 25.

In the *electrically heated boiler* the heating of the water is produced by a coil of wire that is immersed in the water and is heated by a current of electricity. The current must be kept regulated by a rheostat, so that boiling proceeds at a moderate rate.

The chief difficulty is in delivering the steam dry. Condensation is apt to take place in the delivery tube. This can be reduced by inserting a trap in the delivery tube between the boiler and the calorimeter. The trap should, from time to time, be cautiously heated by a Bunsen burner to prevent condensation, but in general, it is better to dispense with the trap and make the exposed part of the

delivery tube as short as possible and carefully cover it with cotton-wool.

If the delivery tube simply passed to a sufficient depth beneath the water, the steam would be delivered at greater than atmospheric pressure, as the pressure of a certain depth of water would have to be overcome. Hence it is better to let the delivery tube pass into a condensing-box immersed in the water. The latter must also be open to the atmosphere by another tube. To prevent any escape of steam by this tube it may be closed by a little cotton-wool. The amount of steam that has been condensed is obtained by weighing the condensing-box (well dried) before it is placed in the calorimeter, and again with the contained water at the end of the experiment. The temperature of the steam is deduced from the barometric pressure. A pressure gauge attached to the boiler affords a means of estimating how far the pressure differs from atmospheric pressure.

For the best results, certain precautions must be observed. The delivery tube must not be connected to the condensing-box until steam has begun to pass freely, and as dry as possible, from the tube. Connection should not be attempted until the temperature of the water has been carefully ascertained and care has been taken that everything is ready for making a deft and prompt connection. After the temperature of the well stirred water in the inner calorimeter has been read every minute for five minutes, the connection is made. The temperature is read every half-minute, the water meantime being kept well stirred by a stirrer (which should be of the same material as the calorimeter and condenser in order to simplify the calculation of the water equivalent). The flame of the ring-burner must be regulated so that the steam does not pass too rapidly. This may be gauged by the rate of the rise of the temperature of the calorimeter, which should not exceed 4° or 5° per minute. In finding the subsequent rate of cooling, the boiler should be disconnected from the condenser and the tube leading to the condenser should be closed by plugs of

cotton-wool to prevent evaporation; but in subsequently weighing the condenser the wool should not be included. The whole determination should be repeated as many times as possible.

A formula for the calculation of the latent heat may be readily worked out. Account must be taken of the water equivalent of calorimeter, condenser, and stirrer. The correction for radiation is made by the method stated on pages 63, 64.

The possible error of the result, so far as it depends on the readings made, should be calculated, and other possible sources of error should be mentioned.

Questions.

1. State the advantages and disadvantages of a rapid flow of steam.
2. Explain why the latent heat should vary with the atmospheric pressure.
3. Must the boiling water be pure? Explain.

XXIII. LATENT HEAT OF VAPORIZATION.

Continuous-flow Method.

Ames' General Physics, p. 269; *Watson's Physics*, §214; *Crew's General Physics*, §287; *Watson, Practical Physics*, §§89–91; *Edser, Heat*, pp. 150–9; *Text-book of Physics (Duff)*, p. 231.

The apparatus for this method may be readily constructed from a Liebig's condenser. Water enters at *D* and leaves at *C* through T-tubes connected to the condenser by short rubber tubes. Superheated steam enters at *A* through a T-tube and the condensed water drops into a covered beaker *E*. The steam is superheated as it flows through a glass tube *FE*. This is first covered with asbestos over which a heating coil of wire* is wrapped, the coil being covered by a second layer of asbestos. *AB* and *FE* are mounted on a wooden frame and *AB* is thickly covered with cotton-wool to prevent radiation. Thermometers T_1, T_3, T_2, T_4,

* "Nichrome" wire (supplied by the Driver-Harris Co., New York) is very suitable.

give the respective temperatures of the superheated steam, the outflowing water, the inflowing water, and the water of condensation. The supply of water may come from the water mains, if this is sufficiently constant in temperature.

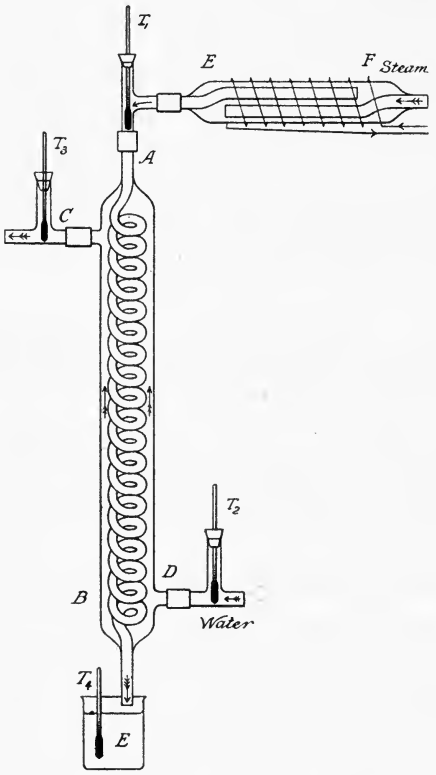

FIG. 26.

It is, however, much better to have a supply of from 5 to 10 gallons in an elevated tank and keep the flow constant by an overflow regulator as indicated in figure 29 (Exp. XXVIII).

The boiler to supply the steam should be large enough to allow of a flow for two hours without refilling (one to two

liters will suffice). The current in the superheating coil should be regulated by a rheostat so that the superheated steam is at about 105°. Some time should be spent in testing adjustments to obtain a suitable current and a rate of flow of water that will give a rise of temperature of about 20°. The tank should be connected with the water service so that it can be readily filled. The water as it comes from the mains will probably be below room temperature and this is an advantage, since with a suitable rate of flow of the steam the water that drops into E will differ but little from room temperature and will suffer little loss of heat by radiation. This will require a proper regulation of the burner that heats the boiler. The burner should be surrounded by a shield of sheet-iron or asbestos to prevent fluctuations caused by air-currents.

The thermometers T_1, T_2, and T_3 should be read once a minute (e. g., T_2 20 sec. after T_1 and T_3 20 sec. after T_2). From the mean of each of these readings, the temperature of E, the mass of water that flows out at C, and the mass of the water that drops into E, the latent heat can be calculated. The specific heat of the superheated steam may be taken as 0.5. A formula can be readily constructed to express the fact that the heat given up by the steam and condensed water equals the heat carried off by the current of water.

Questions.

1. Why does not the water equivalent of the condenser need to be considered?
2. How could you find the amount of error due to conduction of heat from the superheater to the water in the condenser?
3. How could you find the amount of error due to radiation from the condenser?
4. What other sources of possible error are there in this method?

XXIV. THERMAL CONDUCTIVITY.

Edser, Heat, pp. 416–430; *Watson, Practical Physics*, §§106, 107; *Text-book of Physics (Duff)*, pp. 216–220; *Watson's Physics*, §§236–238; *Ames' General Physics*, p. 288; *Crew's General Physics*, §§254, 256.

The *thermal conductivity* of a substance is the amount of heat transmitted per second per unit of area through a plate of the substance of unit thickness, the temperature of the two sides differing by 1° and the flow having become steady. If K be the thermal conductivity, and if a plate of thickness l and area A be kept with one side at a temperature t, and the other at a lower temperature, t', the number of calories that will flow through the plate in time T, after the flow has become steady, will be

$$Q = \frac{KA(t-t')T}{l},$$

whence K can be derived if the other quantities are observed or measured.

Thermal conductivity is in general difficult to measure satisfactorily. The following very simple method cannot be relied on to closer than a few per cent., but it only requires a small portion of the time that the more accurate methods call for.

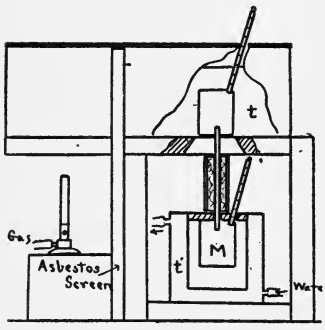

FIG. 27.

A rod or wire of the substance to be tested is inserted at one end into a heavy block of metal, which is heated to a constant high temperature in a bath, through the bottom of which the rod passes. At its lower end the rod is screwed into a heavy block of brass or copper of mass M and specific heat s, which is initially at a very low temperature. Heat is thus conducted by the rod from the bath to the lower block. If the latter neither lost

nor gained heat by convection or radiation, and if there were no losses from the sides of the rod, we could calculate the conductivity of the rod from its dimensions and the mass, specific heat, and rise of temperature of the lower block. The loss of heat from the surface of the rod is almost wholly prevented by enclosing it by a glass tube, which does not come into direct contact with the rod, and wrapping the glass tube with cotton wool and paper.

To allow for radiation or convection to or from the lower block the experiment is modified as follows: The block is enclosed in a vessel surrounded by a water-jacket, through which water at a constant temperature, t', circulates. Now, the rate at which the lower block receives heat through the rod, when the former is at the temperature t', is the mean of the rates at which it receives heat when it is π degrees below t', and when it is π degrees above t'. For let R, R_1, and R_2 represent the rates of conduction of heat (flow of heat in one second) to the lower block at temperatures t', $t'-\pi$, and $t'+\pi$, the upper end being at temperature t. Then

$$R=\frac{KA(t-t')}{l}$$

$$R_1=\frac{KA[t-(t'-\pi)]}{l}$$

$$R_2=\frac{KA[t-(t'+\pi)]}{l}$$

whence

$$R=\tfrac{1}{2}(R_1+R_2)$$

Again, when the lower block is at the same temperature as the jacket, it neither receives heat from nor gives heat to the jacket. And when it is π degrees below it gains heat as rapidly as it loses heat when it is π degrees above. Thus by taking the mean rate as above, the effects of radiation to or from the block are eliminated. In fact, adding a to R_1, to allow for the gain by radiation, and subtracting

a from R_2, to allow for loss by radiation, would leave $R_1 + R_2$ unchanged.

The same would hold true for any other pair of temperatures equidistant from the temperature of the jacket. If the rates of rise at two temperatures equidistant from the temperature of the jacket be r_1 and r_2, by what has been said the rate at the temperature of the jacket would be $\frac{1}{2}(r_1 + r_2)$. Hence, the rate at which the body must be gaining heat is $Ms\frac{1}{2}(r_1 + r_2)$. Hence, by the definition of thermal conductivity,

$$Ms\tfrac{1}{2}(r_1 + r_2) = \frac{KA(t - t')}{l},$$

or,

$$K = \frac{Mls\frac{1}{2}(r_1 + r_2)}{A(t - t')}.$$

The lower block should be cooled initially to about 12° below the temperature of the water that circulates through the jacket by being placed in a bath of ice and water (or snow). When taken out, it must be carefully dried. The jacket may be kept at a constant temperature by water passing and repassing through it between two large vessels, which are alternately raised and lowered about every five or ten minutes. The temperature of the water should be frequently read by a thermometer (which may conveniently pass through a large cork that floats on the surface of the water). If the temperature of the water should show a tendency to rise or fall, a small quantity of cooler or warmer water, respectively, may be added. If the vessels be large and the temperature of the room does not vary widely, there should be no difficulty in keeping the water constant to within .2° for a sufficient length of time.

The hot bath is in the form of a trough, which is heated at one end, while the conducting rod passes into the tank at the other end. To prevent direct radiation from the burner to the rod, thick screens of wood and asbestos are interposed. The temperature of the lower block should be read at least

every minute by means of a thermometer passing through the cork or fiber cover and inserted into a hole in the block, the unoccupied space in the hole being filled with mercury. The readings of temperature will, from various causes, be slightly irregular. They should therefore be plotted in a curve, and the irregularities eliminated by taking the more correct values from the curve. The temperature of the upper block should be read frequently by a thermometer thrust into it. Small quantities of boiling water should be added frequently to the bath to compensate for evaporation.

To gain some idea of the amount of reliance to be placed on the result, the mean rate of rise for each pair of degrees equidistant from the temperature of the jacket should be obtained, and the final mean of all taken in calculating. It is, however, to be noted that the temperatures nearest the jacket temperature should give the best results, since there the radiation is a minimum, and therefore any defect in the method of correcting for radiation a minimum.

(For comparing the conductivities of poorly conducting substances Lees and Chorlton's apparatus is quite satisfactory. Directions for its construction and manipulation are given in Robson's Heat, page 135.)

XXV. THE MECHANICAL EQUIVALENT OF HEAT.

Griffith, Thermal Measurement of Energy, Chap. III; *Edser, Heat*, Chap. XII; *Text-book of Physics (Duff)*, pp. 259–262; *Watson's Physics*, §§250–251; *Ames' General Physics*, pp. 203–205; *Crew's General Physics*, §289; *Rowland, Physical Papers*, pp. 343–476.

The *mechanical equivalent of heat* is the number of units of mechanical energy that, completely turned into heat, will produce one unit of heat, or, in the c. g. s. system, the number of *ergs* in a *calorie*. The apparatus here described is a copy of that used in the University of Cambridge, England, and the following description and introduction is partly taken from that issued to students in that university.

In this apparatus mechanical energy is expended in working against friction, thus producing heat, which is measured by the rise in temperature of a known mass of water.

A vertical spindle carries at its upper end a brass cup. Into an ebonite ring concentric with the cup there fits tightly one of a pair of hollow truncated cones. The second cone fits into the first, and is provided with a pair of steel pins which correspond to two holes in a grooved wooden

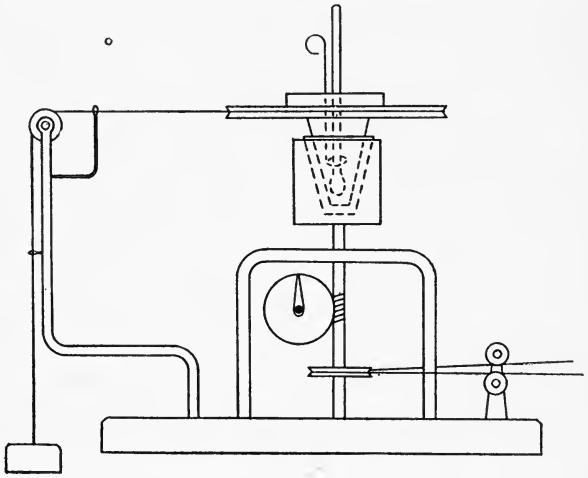

Fig. 28.

disk, which prevents the inner from revolving when the spindle and the outer cone revolve. A cast-iron ring, resting on the disk and fixed by two pins, serves to give a suitable pressure between the cones. A brass wheel is fixed to the spindle, and, by a string passing round this wheel and also round a hand-wheel, motion is imparted to the spindle. A pair of guide pulleys prevents the string from running off the wheel. Above the wheel is a screw cut upon the spindle. This screw actuates a cog-wheel of 100 teeth, which makes one revolution for every 100 revolutions of the spindle.

To the base of the apparatus one end of a bent steel rod is attached; the rod can be fixed in any position by a nut beneath the base. The other end of the rod carries a cradle, in which runs a small guide pulley on the same level with the disk. The cradle turns freely about a vertical axis. A fine string is fastened to the disk and passes along the groove in its edge; it then passes over the pulley and is fastened to a mass of 200 or 300 grams. On turning the hand-wheel it is easy to regulate the speed so that the friction between the cones just causes the mass to be supported at a nearly constant level. To prevent the string from running off the guide pulley, a stiff wire with an eye is fixed to the cradle and the string is passed through this eye. It also passes through an eye fixed to the steel rod, to prevent the weight from being wound up over the pulley.

The rubbing surfaces of the cones must be carefully cleaned and then four or five drops of oil must be put between them; the bearings of the spindle and guide pulley should also be oiled. The cones are then weighed together with the stirrer. The inner cone is then filled to about 1 cm. from its edge with water 2° or 3° below the temperature of the room and the system is again weighed. The cones are then placed in position in the machine and a thermometer is hung from a support so that it passes through the central aperture in the disk and almost touches the bottom of the inner cone.

One observer, X, takes his place at the hand-wheel, and the other, Y, at the friction machine. By working the machine the water is now warmed up until its temperature is nearly equal to that of the room. The index of the counting wheel is read and the temperature of the water is carefully observed every minute for five minutes. Immediately after the last reading, X turns the wheel fast enough to raise the mass until the string is tangential to the edge of the disk. If the string be not tangential the moment of its tension about the axis of revolution is seriously diminished. Y stirs the water and notes the temperature at each passage

of the zero of the counting-wheel past the index; each passage of the zero after the first corresponds to 100 revolutions of the spindle. Y gives a signal at each passage of the zero and X notes the time by aid of a watch. After Y has recorded the temperature upon a sheet of paper previously ruled for the purpose, he also records the time observed by X. After about 1000 revolutions the motion is stopped and the readings of the index of the counting wheel and of the thermometer are recorded. Observations of the temperature are continued every minute for five minutes, the stirring of the water being continued.

The temperature observations are plotted against the time, and the radiation correction is determined as explained on pages 63 and 64. The heat produced is readily calculated from the mass of water, the water equivalent of the cones and stirrer, and the corrected rise of temperature.

From the initial and final readings of the counting wheel and the number of complete revolutions the exact number (n) of revolutions made by the spindle is deduced. The work done is calculated as follows: When the spindle has made n turns the work spent in overcoming the friction between the cones is the same as would have been spent if the outer cone had been fixed and the inner one had been made to revolve by the descent of the mass of M grams. In the latter case M would have fallen through $2\pi nr$ cm. where r is the radius of the groove of the wooden disk, which must be measured. Hence the total work spent against friction and turned into heat is $2\pi nrMg$ ergs. In the report, estimate the possible error of the result as far as it depends upon the errors of observations and measurements.

Questions.

1. What amount of error is due to neglect of the work spent against friction of the bearing of the outer cone?
2. Why must the wheel be turned faster as the experiment proceeds?
3. What effect on the result has the variation of the viscosity of the oil?

XXVI. THE MELTING-POINT OF AN ALLOY.

Robson, Heat, pp. 77–79; *Findlay, Phase Rule*, pp. 220–223; *Ewell, Physical Chemistry*, pp. 271–272.

If an alloy is melted and is allowed to cool while its temperature is continuously observed, and a curve be then drawn with times as abscissæ and temperature as ordinates, it will be found that at certain points the curvature abruptly changes, the fall of temperature being decreased or even ceasing. At the moment corresponding to such a point, the alloy is radiating heat to the room, and the fact that its temperature does not fall as rapidly indicates that heat is being produced internally by some change of state of the material. Such a point is therefore a solidifying-point of some constituent of the alloy or of the eutectic alloy.

The assigned* metals are carefully weighed and melted in an iron cup. A copper-constantin† thermocouple is plunged into the liquid metal and kept there until the entire mass is solid. A porcelain tube should cover one wire for some distance from the junction. The terminals are connected to a calibrated galvanometer through a resistance such that the maximum deflection will keep on the scale. The galvanometer is read every half-minute and the time of each reading is noted. When the readings are commenced, the metal should be considerably above the melting-point and the readings should be continued for some time after the metal is apparently solid. For calibration of the galvanometer, see Exp. LVIII.

Plot the galvanometer deflections against the time. Determine the electromotive force corresponding to the galvanometer deflections where the curvature changed, and from the constants of the thermocouple, or a chart giving the

* Tin and lead are suitable metals. The changes of curvature are more distinct if the former is in excess. The eutectic of tin and lead is composed of 37% lead, 63% tin, and melts at 182.5° (Rosenhain and Tucker, Roy. Soc. Phil. Trans., 1908, A. 209, p. 89).

† "Advance" Wire, Driver-Harris Co., Harrison, N. J.

temperature for different electromotive forces, determine the temperature of these points.

Tabulate the observed temperatures of these transition points and your opinion of what they represent.

Questions.

1. Explain why the second transition point is represented by a horizontal portion of the cooling curve while the first transition point is merely represented by a change of curvature.
2. Will the temperature of the first point vary with the initial concentration?
3. Will the temperature of the second transition point vary with the initial concentration?

XXVII. HEAT VALUE OF A SOLID.

Ferry and Jones, pp. 237–242.

(A) Hempel Bomb Calorimeter (*Constant-volume Calorimeter*)

A pellet of the fuel to be tested is formed in a press, a cotton cord being imbedded with a loose end. After being pared down to about 1 gm. and brushed, it is carefully weighed. It is then suspended in a Hempel combustion bomb, and the thread is wrapped around a platinum wire connecting the platinum supports of the basket. The terminals attached to these supports are connected with several Edison or storage cells sufficient to just bring the wire to a brilliant incandescence (as ascertained by a preliminary trial).

The bomb is charged with oxygen under at least fifteen atmospheres' pressure, either from a charged cylinder or produced by a retort. Bomb and pressure gauge should be immersed in water while the oxygen is being supplied. Ascertain that the bomb valve is open and that all connections are screwed tight. Open the cylinder valve (if a cylinder is used) until the pressure becomes high, and then close. Lift the bomb out of the water, loosen one of the connections, and allow the mixture of air and oxygen to escape; then

tighten, replace in water, open the cylinder valve again until the pressure becomes high (at least fifteen atmospheres); close both the cylinder valve and that of the bomb, and finally disconnect and dry the bomb. (If the oxygen is produced in a retort, partly fill the latter with a five to one mixture of potassium chlorate and manganese dioxide, connect to the bomb and pressure gauge, and heat the upper part slowly with a Bunsen burner.)

Attach the electrical terminals, place the bomb in the special vessel containing about a liter of water, adjust the Beckmann thermometer to read about 1° (see p. 65), stir the water continually, and read its temperature every half-minute for five minutes, estimating to tenths of the smallest graduation. Close the electric switch; after a few seconds open it and read the temperature of the continually stirred water for ten minutes.

Let H be the heat value of the fuel and m the mass of the specimen, M the mass of the water, e the water equivalent of the bomb, t_1 the initial temperature, and t_2 the final temperature (corrected for radiation, see p. 63.) Then

$$H = \frac{(M+e)(t_2-t_1)}{m} \text{ cal. per gm.}$$

The best method in practice to determine e is to repeat the determination, using salicylic acid as fuel, and assuming its heat value to be 5300 calories per gram.

(B) Rosenhain's Calorimeter (*Constant-pressure Calorimeter*)

Phil. Mag., VI, 4, p. 451.

Instead of burning the fuel in a fixed volume of highly compressed oxygen, the oxygen is supplied continuously at only slightly above atmospheric pressure.

The coal is pulverized and a sample is compressed, in a special screw press, into a pellet weighing about one gram. This is placed on a porcelain dish which rests on the bottom

of the inside chamber. The ignition wire should be about 3 cm. of No. 30 platinum wire and the external terminals should be connected to storage-battery terminals through a key and a resistance such that the wire will glow brightly. A gasometer is charged with oxygen from a cylinder or generated from "oxone" and water. The action of the different valves having been studied, the apparatus should be assembled, the upper side valve (see Fig. 29) being closed and the ball valve lowered. Connect with the oxygen supply through a wash-bottle, turn on a very gentle stream of oxygen, and pour into the outer vessel a measured volume of water, at the room temperature, so that the combustion chamber is just covered.

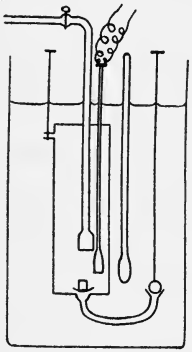

FIG. 29.

If a Beckmann thermometer (see p. 65) is used, adjust to read between 0° and 1° in this water. The bulb should be supported on a level with the center of the combustion chamber. Read the temperature every half-minute for five minutes, then increase the oxygen current, and carefully read the temperature and the time, close the key and ignite the pellet with the hot platinum wire, and *immediately remove the wire.* During such operations it is best to hold the inner vessel steady by grasping the oxygen inlet tube. Keep the water pressure in the gasometer constant, and as combustion proceeds increase the flow of oxygen. If possible, read the thermometer every half-minute.

When combustion has ceased, move the hot wire about to ignite any unconsumed particles. Keep the wire hot as short a time as possible and *remove it immediately from any combustion,* otherwise it is liable to be melted.

Finally, turn off the oxygen supply, open the upper valve, and raise the ball valve, allowing the water to enter the inner chamber. Then force out the water by closing both valves and turning on the oxygen. Record the highest

temperature and the time and the temperature every half-minute for five minutes. For radiation correction, see p. 63, and for formulæ, see (A) preceding.

To determine *e*, assemble the apparatus, including the Beckmann thermometer, and pour in 1000 c.c. of water. Determine very carefully the temperature with a 0.1° thermometer and then add 500 c.c. of water at about 50°, the temperature of which has also been very carefully determined. Determine also very carefully the final steady temperature and from these data determine *e*.

For anthracite coal add sugar in the proportion 3:1. (Heat of combustion of sugar = 3900 calories per gram.)

Questions.

1. Calculate the heat value of (a) one kilo of this substance, (b) one short ton in B. T. U. per lb., (c) the mechanical energy equivalent to the latter.

2. What error would be caused by (a) an error of 20 in the water equivalent? (b) allowing a current of 5 amperes to flow through a platinum wire of 2 ohms' resistance for 5 seconds? (c) Neglecting the radiation correction?

XXVIII. HEAT VALUE OF A GAS OR LIQUID.

Ferry and Jones, pp. 243–246.

The *heat value* will be determined with Junker's calorimeter.

(A) A measured volume (*v* liters) of gas under an observed pressure, *p*, is burned in the calorimeter, and the rise of temperature, from $t_1°$ to $t_2°$, of a mass of *M* gr. of water, is determined. The flow of water and gas is so regulated that the burned gas leaves the calorimeter at approximately the temperature of the entering gas, and there should be a difference of at least 6° in the temperature of the in- and outflowing water. Also, the flow of water must be sufficient to furnish a constant small overflow at the supply reservoir (see Fig. 30). The burner should be lighted outside the calorimeter. When the temperatures indicated on the various thermometers have become constant, note the gasometer reading, and immediately collect in graduates the

heated overflowing water, and also the water condensed by the combustion of the gas. Let the mass of the latter be m gr., and its temperature t'. Note the temperatures of the inflowing and outflowing water every 15 sec. until two or three liters have passed through. Then immediately note the gasometer reading and remove the graduates.

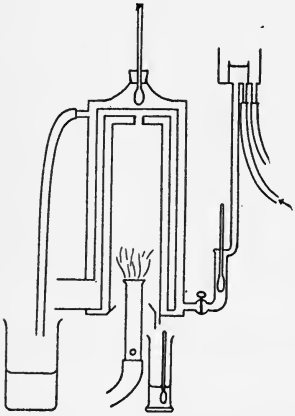

FIG. 30.

Assuming that condensation of the gas occurs at 100°, the heat liberated is $m\,[536+100-t']$. If H represents the heat value of the gas in gram-calories per liter and v the volume, reduced to 0°, and 760 mm.,

$$H=\frac{M(t_2-t_1)-m(636-t')}{v}.$$

(B) To determine the heat value of a liquid fuel, the gas burner is replaced by a suitable lamp which is attached to one arm of a balance. The rate at which the liquid is consumed is determined from the weights in the pan, on the other side, at different times. It is best to make the weight in the pan slightly deficient and note the exact time when the balance pointer passes zero, as the liquid is consumed. Practically complete combustion is obtained with a "Primus" burner, supplied from a reservoir where the liquid is under considerable pressure. With very volatile liquids, the opening of the burner must be large and the pre-heating tubes must be in the cooler part of the flame.

Express your results in (*a*) calories per liter (*b*) B. T. U. per gallon or cubic foot.

Questions.

1. Is the heat value of a gas, in calories per gram, definite? Per liter?
2. What difference would there be in the result if all the water vapor escaped without condensing?
3. Why is no radiation correction necessary?

XXIX. PYROMETRY.

Edser, Heat, pp. 339–411; *Bulletin, Bureau of Standards*, I, 2, pp. 189–255; *Watson, Practical Physics*, §§208–210; *Le Chatelier, High Temperature Measurements*, Chaps. III, VI, IX.

This exercise is a study of three of the methods used in the measurement of very high temperatures.

A hollow black body is to be heated electrically and the temperature of its interior is to be determined by means of a calibrated thermocouple. A platinum resistance thermometer is to be calibrated, and also an incandescent lamp is to be calibrated for use as an optical pyrometer.

Electric Furnace.—The electric furnace (see Fig. 31) consists of a thin porcelain cylinder about 15 cm. long and

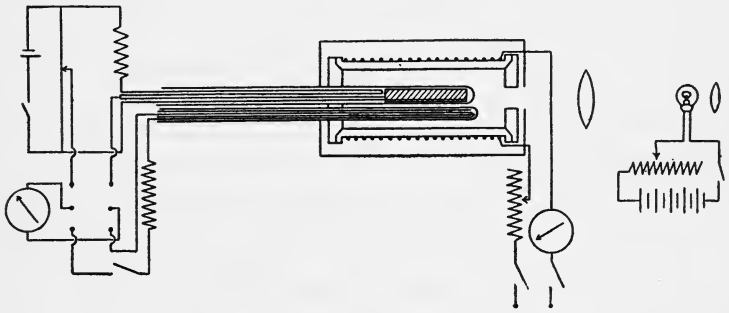

FIG. 31.

10 cm. in diameter upon which is wound about 5 m. of No. 22 "Nichrome" wire,* if a 220-volt supply is to be used. Whatever the voltage, the winding must be such as to consume about half a kilowatt. The ends of the cylinder are closed by porcelain caps with proper apertures and the whole is surrounded by many layers of asbestos.

Heating and cooling must be very gradual so that the thermocouple and platinum thermometer may acquire the temperature of the furnace. The highest temperature should not exceed 1000°.

*See note bottom of page 99.

Thermocouple.—A platinum and platinum +10% rhodium thermocouple should be connected to a galvanometer through a key and such a resistance as will keep the deflection on the scale at the highest temperature. The galvanometer *with resistance* should be calibrated as a voltmeter (Exp. LVIII). The chart or table accompanying the couple gives the temperature of the hot junction when the electromotive force is known. (The cool junction should be in ice and water.)

Platinum Resistance Thermometer.—The platinum resistance thermometer consists of a coil of fine platinum wire (for example, 50 cm. of No. 30) wound on a porcelain frame and surrounded by a glazed porcelain tube. The coil constitutes one arm of a Wheatstone's bridge. A pair of dummy leads are connected to an adjoining arm (see figure) and compensate for the heating of the lead wires. A suitable switch connects the galvanometer to either the bridge or the thermocouple.

Optical Pyrometer.—The optical pyrometer consists essentially of a lens and a miniature incandescent lamp. The lens focuses the interior of the enclosed furnace (an ideal black body) on the filament of the lamp. The current through the filament is adjusted until the tip of the filament is invisible against the image of the furnace. When this is true, both must be emitting similar light, and therefore they must be at approximately the same temperature. A small eye-piece aids in observing the tip of the filament. The incandescent lamp filament is to be calibrated; i. e., the current necessary to heat the tip of the filament to different temperatures is to be determined. The temperature of the filament is determined by finding the temperature of the furnace, by the thermocouple, when the two have the same temperature. (The incandescent lamp circuit contains an ammeter for measuring the current, which is omitted from Fig. 31.)

Observations.—While the furnace is *slowly* heating, and also while it is *slowly* cooling, observe at frequent intervals,

(A) the galvanometer deflection with the thermocouple, and the resistance in the galvanometer circuit; (B) the resistance of the platinum thermometer; (C) the current through the filament. The three observations should be made in succession and the times of each recorded. (D) Calibrate the galvanometer as described in Exp. LVIII, if the constant is not furnished. (E) If time permits, use the optical thermometer to determine the temperature of various distant, brightly heated bodies; e. g., melted silver, iron, or copper. An image of the hot body is formed upon the tip of the filament, and the current through the latter is adjusted until the two are indistinguishable. The temperature corresponding to this current is obtained from the calibration (see (e) below).

Report.—(a) Tabulate readings. (b) Plot the three sets of readings against the time. (c) Make a second plot with resistance as abscissæ and, for ordinates, the *platinum temperatures*, as given by Callender's equation,

$$pt = 100\frac{R_t - R_0}{R_1 - R_0}.$$

where R_0 is the extrapolated resistance at 0° and R_1 is the resistance at 100°. Transfer to this plot also the readings of the true temperature, t, and determine the mean value of Callender's difference constant, δ, by applying at several points the equation

$$t - pt = \delta\left(\frac{t}{100} - 1\right)\frac{t}{100}.$$

For pure platinum δ is 1.50. The platinum resistance thermometer is the most accurate, convenient method of measuring temperature below 1000°.

(d) Construct a curve which gives the temperature of the tip of the incandescent lamp filament plotted against the current. (e) Finally, determine by the latter curve the temperatures of any bodies tested with the optical pyrometer and record the results. (For a discussion of the error in

assuming for different bodies that the radiation is similar to that from a black body, see the above reference to the Bulletin of the Bureau of Standards and Haber, "Thermodynamics of Gas Reactions" pp. 281–291.)

Questions.

1. Would you expect the platinum resistance thermometer to attain a slightly higher or a slightly lower temperature than the thermocouple? Explain.

2. Is any correction required for the absorption of the lenses in such an optical pyrometer? Explain.

SOUND.

XXX. THE VELOCITY OF SOUND.

Text-book of Physics, (*Duff*), pp. 319, 320, 334–337; *Watson's Physics*, §§287, 288, 309; *Ames' General Physics*, pp. 337, 363; *Crew's General Physics*, §213; *Poynting and Thomson, Sound*, Chap. VII.

The *velocity of sound* in a medium can be found if the wave-length, L, in the medium of a note of frequency n can be determined; for $v = n\ L$. If the medium be contained in a tube, one end of which is closed, the closed end must be a node and the open end a loop. Hence the length of the tube must be an odd number of quarter wave-lengths. Such a tube will resonate to a fork, if the wave-length of a natural vibration of the pipe be the same as the wave-length to which the fork gives rise. Thus, if the length of pipe that resonates to a fork of known pitch be measured, we have the means of finding the velocity of sound.

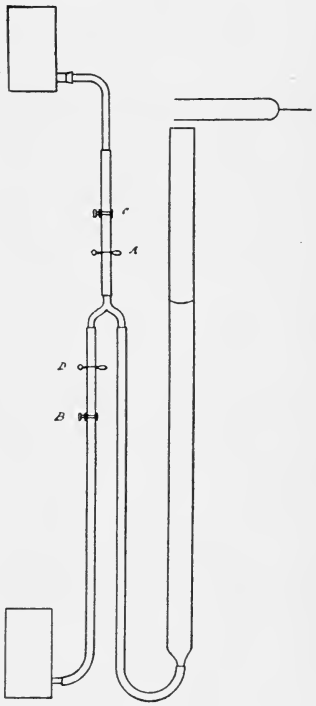

FIG. 32.

A long glass tube is mounted on a stand. Water is introduced from the bottom, where is attached a rubber tube provided with a pinch-cock and connected to a glass bottle. By raising and lowering the bottle, water may be brought to any height in the tube. The additional connections represented in figure 32, permit raising or lowering the water level by opening pinch-cocks. When A is opened

the tube fills and the tube empties upon opening *D*. *C* and *B* are pinch-cocks which regulate the rate of flow. The vessels are so large compared with the capacity of the tube that only rarely is it necessary to transfer water from the lower to the upper vessel.

If a tuning-fork be vibrated above the tube, resonance will first take place when the air column is approximately one-quarter wave-length of the fork, next when three-quarter wave-lengths, etc. In reality, the first loop is not exactly at the open end of the pipe, but a short distance beyond the open end. The distance between two nodes is accurately half a wave-length, and it is from this distance that the wave-length is best determined.

The tuning-fork may be sounded by gently striking the end of the fork against the knee or a block of soft wood. The fork should be held above the end of the tube, so that the plane of the prongs includes the axis of the tube. Each node should be located very carefully, at least four times, each location being tested both with the water rising and with the water falling, and the distance of each position from the open end noted. The mean is taken as the true distance. The whole should then be repeated with a fork of different pitch. Observe the temperature and barometric pressure, and measure the diameter of the tube. With a little practice one can often locate nodes corresponding to the higher modes of vibration of the fork. This should be tried and, from their wave-lengths and the velocity of sound as already determined, the pitch of these higher sounds can be calculated.

The pitch of the forks used may be determined by comparison with a standard fork by the *method of beats*. The standard is mounted on a resonance box and is set in vibration by pulling the prongs together with the fingers and then releasing them. The fork of unknown pitch is sounded in the usual way, and the end of the shank is set upon the resonance box of the standard. If the two forks are of nearly the same pitch, beats will be heard. With a stop-

watch the time of ten, fifteen, or twenty beats, as may be convenient, is several times determined. Dividing by the time, we have the number of beats per second, and this is the difference of pitch of the two forks. To determine which fork is the higher, add a little wax to one prong of the fork used in the experiment. Since this increases the inertia of the fork, it decreases its pitch. If originally the two forks have very nearly the same pitch, so that there is only a fraction of a beat per second, very small amounts of wax should be added. A large piece of wax might change the pitch of the fork from above that of the standard to below. (Time may often be saved by comparing the fork with the standard during the necessary delays of the work described below.)

The velocity of sound in carbon dioxide may be determined in a similar manner. The water surface is first lowered to the bottom of the tube and the tube is filled with the gas from a generator through a small tube lowered just to the water surface (not below). The generator consists of a tube filled with marble, surrounded by dilute hydrochloric acid. In filling the generator with marble, use only whole pieces, carefully excluding any dust or pieces small enough to drop through into the outer vessel. If the gas is not evolved in sufficient quantity, add hydrochloric acid to the outer vessel. When the air in the tube has been entirely displaced by the gas, a lighted match introduced into the top of the tube will be extinguished. The delivery tube is now withdrawn from the resonance tube, while the gas still flows out to fill the volume occupied by the delivery tube.

The water surface is now slowly raised and the nodes located for one of the forks. Observations can only be made with the water rising, for, when the water surface is lowered, air enters the tube. Hence each node can be located but once. The tube is again filled and the nodes redetermined with the same fork.

From the distance between the nodes and the pitch of the fork the velocity of sound is determined.

For each gas, find the correction for the open end; that is, the displacement of the loop beyond the end of the tube. Use the mean position of the highest node and one-fourth of the mean wave-length. Find what fraction this displacement is of the radius of the tube.

Calculate, from the mean values of the velocities, the velocity of sound in each gas at 0° C. (see references). Find also the ratio of the specific heats from the velocity at 0° C., the standard barometric pressure (both in absolute units) and the density as given in Table VI.

Questions.

1. Explain why (a) readings are made with both rising and falling water (b) the plane of the prongs of the fork must contain the axis of the tube.
2. What is the influence of atmospheric moisture upon the velocity of sound?

XXXI. VELOCITY OF SOUND BY KUNDT'S METHOD.

Text-book of Physics (*Duff*), p. 338; *Watson's Physics*, §317; *Ames' General Physics*, p. 364; *Crew's General Physics*, §215; *Poynting and Thomson, Sound*, pp. 115–117; *Watson's Practical Physics*, §113.

A glass tube, *A G*, about a meter long and about 3 cm. internal diameter is closed at one end by a tight-fitting piston, *C*, and at the other end by a cork through which passes a glass tube having at one end a loosely fitting cardboard disk, *D* (Fig. 33). The glass tube should be about a meter long. A little dry powdered cork is sprinkled in the tube, the stopper at *G* is loosened, and a current of air, dried by passage through several drying tubes, is slowly forced through the hollow rod of the piston, *C*. The stopper at *G* is then replaced and the glass tube, *F*, is held at the center and stroked longitudinally with a damp cloth. The piston, *C*, is adjusted until the powder collects in the sharpest attainable ridges. These ridges will appear where the pressure changes are least; that is, at the loops. Measure carefully the distance between two extreme ridges and

divide by the number of segments into which the tube is divided. This distance (between two loops) is a half wave-length of the waves in the tube. Disturb the powder and make a new adjustment of the piston, C, and a new measurement of the half wave-length. Make a third repetition of the adjustments and readings.

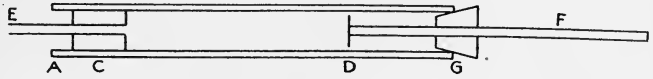

FIG. 33.

Fill the tube with another dried gas, for example, carbon dioxide, illuminating gas, hydrogen, oxygen, or hydrogen sulphide, and determine the half wave-length. If n is the constant pitch of the note emitted by the glass tube and l is the wave-length in the gas

$$v = nl \quad \therefore \frac{v_1}{v_2} = \frac{l_1}{l_2}.$$

Since the velocity changes at the same rate with change of temperature in all gases (see references), the velocity of sound or compressional waves at zero degrees in any other gas than air can be calculated from the ratio of the wave-lengths at a common temperature, and the velocity in air at zero degrees (33,200 cm. per second).

From the velocity of sound at zero degrees in the gases other than air and the standard atmospheric pressure calculate the ratio of specific heats, γ (see references). Table VI gives the densities of the more common gases and vapors at zero degrees and a pressure of 76 cm. of mercury = 1013200 dynes per square centimeter.

Questions.

1. Calculate (a) the velocity of compressional waves in glass, (b) the elasticity E. (Notice that each end of the glass rod must be a loop, and the center a node. The density of glass can be obtained from Table VIII.)
2. Why must the glass rod be set in *longitudinal vibration*?
3. Why does the powder collect at the loops?

LIGHT.

27. Monochromatic Light.

The simplest and most useful monochromatic light is the *sodium flame.* Sodium may be introduced into a Bunsen flame by surrounding the tube of the burner with a tightly fitting cylinder of asbestos which has been saturated with a strong solution of common salt and formed into cylindrical shape by wrapping around the burner while still damp. As the top of the cylinder is exhausted, it should be torn off and the rest of the tube pushed up into the lower part of the flame. A piece of hard-glass tubing held in the flame will also give a good sodium light.

Elements giving red, green, blue, and violet light will be found in Table XVIII. Salts of these elements (e. g., KNO_3, $SrCl_2$, $CaCl_2$, $LiCl$) may be introduced into the outer edge of a bunsen flame, either in a thin platinum spoon, on copper gauze, or by a piece of wood charcoal which has absorbed a solution. If a very intense light is not required, a vacuum tube is a very satisfactory source (Table XVIII). Intense light of one general color may be obtained by filtering sun light or the light from an arc light through colored glass or gelatine. The solutions given in the accompanying table give much purer monochromatic light.

Light Filters (Landolt).*

Color	Thickness of layer (mm.)	Aqueous solution of	Grams per 100 c.c.	Average wave-length (Angstrom units)
Red..	20 20	Crystal violet 5BO Potassium chromate	.005 10.	6560
Green	20 20	Copper chloride Potassium chromate	60. 10.	5330
Blue..	20 20	Crystal violet Copper sulphate	.005 15.	4480

* *Mann, Manual of Advanced Optics,* p. 185.

28. Rule of Signs for Spherical Mirrors and Lenses.

Mirrors.—Consider the side upon which the incident light falls as the positive side of the mirror. If the object, the image, or the principal focus is on this side, their respective distances, (u, v, $f=r/2$) will be positive; if on the other side, negative. Therefore, the focal length (and hence radius) is positive for concave mirrors and negative for convex. The object distance, u, will obviously, in most cases, be positive.

The formula for all spherical mirrors is:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}=\frac{2}{r}$$

if the signs of the numerical quantities which are substituted for u, v, f, and r are determined by the above rule.

Lenses.—Let all the distances, u, v, f, r_1, r_2 be positive for the *double convex lens*, when the object is outside the principal focus; that is, in the most common case. The formula for all lenses is then

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}=(n-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

As an illustration of the application of this rule, consider the signs of these distances when an image of a real object is formed by a double concave lens. The distance, u, of the object is obviously measured on the same side as it would be in the standard case of the double convex lens and is, therefore, positive. The distances f and v are, however measured on the same side of the lens as the object, or opposite to the standard case with the double convex lens, and are, therefore, negative. r_1, the radius of the front face, is on the same side as the object, while in the case of the double convex lens this radius is on the other side, therefore r_1, and similarly r_2, is negative for a double concave lens.

XXXII. PHOTOMETRY.

Text-book of Physics (*Duff*), p. 353; *Watson's Physics*, §§361–364; *Ames' General Physics*, pp. 437, 442; *Watson's Practical Physics*, pp. 382–387; *Edser, Light*, pp. 9–20; *Stine's Photometrical Measurements; Palaz' Photometry.*

The *intensity of illumination* of a surface by a source of light of small area varies inversely as the square of the distance. Hence it follows that, if two lights produce equal intensities of illumination at a point, P, their *illuminating powers*, or the intensities of illumination they can produce at unit distances, are directly as the squares of their distances from P. This is the basis of all practical methods of comparing illuminating powers.

As a means of testing when two different sources of light produce equal illumination at a point, various so-called screens have been used. The one that has been most extensively employed is *Bunsen's grease-spot screen.* It is based on the fact that a grease-spot on paper is invisible when the paper is equally illuminated on both sides, since viewed from one side as much light is gained by transmission from the farther side as is lost by transmission to the farther side.

Another screen more perfect in some respects is that of *Lummer and Brodhun.* A white opaque disk (see figure 34) is illuminated on opposite sides by the two sources of light. An arrangement of mirrors and lenses enables one eye to view both sides at once. Two plane mirrors reflect rays from the two sides into a double glass prism. This consists of two separate right-angled prisms, the largest face of one being partly beveled away and the two being cemented together by Canada balsam, which has the same optical density as the glass, and therefore reflects no light. The central rays from the left pass through the double prism to the tele-

scope while the marginal rays are totally reflected by the beveled edge. The marginal rays from the right are totally reflected and reach the telescope, but the central rays pass through. Thus the eye sees a circular portion of the left side of the opaque disk and a surrounding rim of the right side.

To eliminate error from lack of symmetry, the lamps compared should be interchanged in the course of the readings or the screen should be rotated 180°.

The lights to be compared are mounted at opposite ends of a graduated bar 3 meters long, which, with a parallel bar and suitable supports, constitutes the photometer bench. The screen is mounted on a carriage movable along the bench.

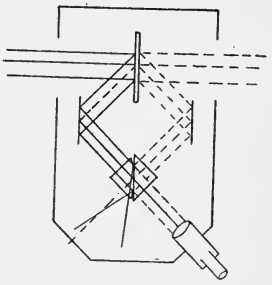

Fig. 34.

Many light standards have been employed. A candle of certain carefully specified dimensions was long employed, and the illuminating power of such a candle is still regarded as the unit and called "one candle-power," but, for practical purposes in testing, some other standard is usually employed. The best such standard is a lamp, with a wick of specified form and dimensions, burning amyl acetate with a flame of specified height. (See references.) Its relation to the "candle-power" is 1 c. p. = 1.14 amyl acetate units. For most purposes an incandescent lamp that has been standardized is the most useful standard, especially in the study of incandescent lamps; but it must not be used any great length of time without being re-standardized, since its illuminating power changes with prolonged use.

The chief difficulty in comparing two different forms of light is due to the fact that a difference of quality of the two lights renders perfectly equal and similar illumination of the two sides of the screen impossible. This difficulty is still more marked in the study of arc-lights (for mechan-

ical arrangements see *Stine*, p. 236), for which it is best to use as an intermediate unit a very powerful incandescent lamp. The latter may be standardized by comparison with an ordinary incandescent lamp, which again is compared with an amyl acetate standard. *Before connecting a lamp to a circuit, ascertain that the voltage is not excessive for that particular lamp.*

(A) Carefully *standardize an incandescent lamp*, for use as a working standard, by comparison with either an amyl acetate lamp or a standardized incandescent lamp. If the latter is used, the lamps should be in parallel, that the voltage may be the same, and a variable resistance should also be in the circuit, by varying which the voltage across the lamps is maintained at the value prescribed for the standard. See that the filament of the standard is in the marked azimuth and note the position of the filament for which the other lamp is standardized.

In each case several settings of the screen should be rapidly made, and then the screen reversed and several more made. The calculations may be facilitated by Table XXI.

(B) *The law of inverse squares* should be tested by comparing two somewhat different incandescent lamps (1) when 3 m. apart, (2) when 2.5 m. apart, (3) when 2 m. apart on the photometer bench. The ratio of their illuminating powers, as deduced in the three cases, should be a constant.

(C) *The horizontal distribution* of candle-power about an incandescent lamp should be studied. This incandescent lamp should be connected in parallel with the working standard and the voltage maintained at the value for which the latter was standardized. The lamp should be mounted on the revolving lamp-holder of the photometer, care being taken to have the center of the filament at the same height as the center of the screen. The lamp is first turned to *the standard position*, i. e., the position in which the plane of the shanks of the filament is at right angles to the photometer bench, and a *marked face* of the lamp is toward the screen. The candle-power of the lamp is to be found in

this position and at positions 30° apart as the lamp is rotated through 360°. Two careful readings should be made at each angle and the c. p. deduced from the mean. The mean of all these values of the c. p. gives the *mean horizontal candle-power*. A curve should be plotted, giving the distribution of c. p. in *polar co-ordinates*. The mean horizontal candle-power is more easily determined by continuously rotating the lamp about a vertical axis by means of a small motor.

(D) *Efficiency of an Incandescent Lamp.*—Keeping the potential of the working standard at the proper point (or calibrating and using a lamp which may be connected to the lighting circuit if the potential of that is constant), apply various potentials to the lamp used in (C) at intervals between about 25% below the normal voltage to 25% above. For each potential, determine the candle-power and current. Calculate the watts consumed and the watts per candle-power.

In the report, plot in three curves, with volts as abscissæ, (a) current, (b) candle-power, (c) watts per candle-power. The scales of the three curves should be shown on the vertical axis.

(*E*) *Mean Spherical Candle-power.*—With the lamp in the standard position of (C), find the c. p. at intervals of 30° in a vertical circle by rotating the lamp about a horizontal axis. After this, start again from the standard position and first turn the lamp through 45° in azimuth (or around a vertical axis), and then, as before, find the c. p. at intervals of 30° in the vertical circle of 45° azimuth, and so for the vertical circles of 90° and 135° azimuth. As before, plot the curves of distribution in polar co-ordinates.

To find the mean spherical candle-power omit any repetitions and take the mean of the readings in the following positions:

1. At tip	1
2. At 60°, 120°, 240°, 300° on the vertical circles of 0° and 90° azimuth	8
3. At 30°, 150°, 210°, 330° on the vertical circles of 0°, 45°, 90°, 135° azimuth	16
4. 12 equidistant positions on horizontal circle	12
5. At base (o)	1
Total,	38

These directions are chosen because they are nearly uniformly distributed in space.

(F) If time permit, study the differences of quality of light given by different sources; e. g., compare an oil lamp and an incandescent lamp using interposed colored glasses: (1) a pair of red glasses, (2) of yellow glasses, (3) of blue glasses. Calculate the relative illuminating powers in each case.

The possible error may be deduced as usual from the mean deviation of the readings in a set.

Questions.

1. Explain the deviation of the current-voltage curve from a straight line.
2. What is the advantage in increasing the voltage applied to an incandescent lamp? Disadvantage?

XXXIII. SPECTROMETER MEASUREMENTS.

Text-book of Physics (*Duff*), pp. 387, 436, 437, 440; *Watson's Physics*, pp. 468, 469, 493; *Ames' General Physics*, pp. 459, 460, 505, 506; *Crew's General Physics*, p. 510; *Edser's Light*, pp. 86-91.

A *spectrometer* consists of a framework supporting a telescope and a collimator, both movable about a vertical axis, and a platform movable about the same axis. The platform is for supporting a prism or grating. The collimator is a tube containing an adjustable slit at one end and a lens at the other end. The purpose of the collimator is to render light coming from the slit parallel after it leaves the lens. (Only when the light that falls on a prism is parallel light, that is, light with plane wave front, does it seem when emerging from the prism to come from a clearly defined source. When it is not parallel, there is spherical aberration.) Hence the slit of the collimator should be in the principal focus of the lens. The telescope is for the purpose of viewing the light that comes from the collimator, either directly or after the light has been refracted or reflected. Hence, since the light that comes from the collimator is supposed to be parallel, that is, as if it came from a very distant source, it follows that if the telescope is to

receive the light and form a distinct image of the slit, the telescope must be focused as for a very distant object (theoretically an infinitely distant one).

The first adjustment is to focus the telescope. First focus the eye-piece of the telescope on the cross-hairs and then focus the whole telescope on a distant object out of doors. The telescope will now be in focus for parallel rays. Turn the telescope to view the image of the slit formed by the collimator and adjust the slit until its image is seen most distinctly.

That the instrument should be in complete adjustment, it is necessary that the telescope, collimator, and platform should rotate about the same axis, and that the optical axes of the telescope and the collimator should be perpendicular to this axis of rotation. For fine work spectrometers are made with all these parts separately adjustable, but simpler instruments have the telescope and collimator put into permanent adjustment by the instrument maker. In any case the telescope and collimator should not be adjusted for level without the advice of an instructor.

Adjustment of Prism.—The refracting edge of the prism must be made parallel to the axis of the instrument. Place the prism on the platform with one of the faces perpendicular to the line joining two of the leveling screws. Turn the collimator slit horizontal and place the telescope so as to receive the image of the slit reflected from this face of the prism. Adjust these two leveling screws until the image of the stationary edge of the slit coincides with the horizontal cross-hair. Then observe the image reflected from the other face and adjust the third leveling screw until the edge of this image is on the horizontal cross-hair. A little consideration will show that, when these two adjustments have been made, both faces, and therefore the refracting edge, are parallel to the axis. Restore the collimator slit to the vertical position.

Measurement of the Angle of a Prism.—Method (A).—The prism should be so placed that the faces are about equally

inclined to the collimator. To secure good illumination, the edge of the prism should be near the axis of the instrument. The telescope is turned to view the image of the slit in the two faces alternately, and the scale and vernier read when the slit and cross-hair coincide, the slit being narrowed until barely visible. If the scale is provided with two verniers, to eliminate error from eccentricity, always read them both. Half of the angle between the two positions of the telescope gives the angle, A, of the prism, as may be readily seen by drawing a diagram. The readings on each side should be repeated three times.

Method (*B*).—The following method, which is sometimes easier than the preceding, may be used if the platform that carries the prism can be rotated and the rotation read by a scale. Turn one face of the prism so as to reflect the image of the slit into the telescope. Adjust the telescope until the vertical cross-hair coincides with the slit and then read the platform scale. Now rotate the platform until the other face of the prism reflects the slit and again read the platform scale. The difference of the readings is $180 \pm A$, as may readily be seen by drawing a figure. The observation should be repeated at least three times.

We are now in a position to make a final measurement for finding the *index of refraction* of the glass of the prism for any particular light of the spectrum, for instance, sodium light (see p. 124). The only additional measurement necessary is the deviation produced by the prism when it is in such a position that it gives a minimum deviation to the light refracted through it.

Minimum Deviation.—The position of minimum deviation is such that the image of the slit seen in the telescope moves in the same direction (that of increasing deviation) no matter which way the platform carrying the prism is turned. There are, of course, two positions in which the deviation can be obtained, one with the refracting edge turned toward the right of the observer, and the other with it toward the left. The deviation in each case is the angle

between the corresponding position of the telescope and its position when looking directly into the collimator, the prism being removed. But it is not necessary to remove the prism, for it is easily seen that the minimum deviation must also be equal to half of the angle between the two positions of the telescope when observing the minimum deviation.

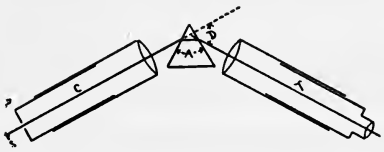

FIG. 35.

These two positions should be observed three times successively, and the mean value for the minimum deviation, D, taken. From A and D the index of refraction may be calculated by the formula

$$n = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A}.$$

If time permit, determine the index of refraction for as many other wave-lengths (colors) as possible (see p. 124).

The possible error of the determination of the refractive index can be calculated by means of formulæ deduced by the calculus, as explained on pp. 7, 8. A simple, but less accurate method is to recalculate n with A and D, increased by their mean deviations and to consider the difference between this value and the original value as the possible error.

It is probable that in this experiment there are other sources of error that exceed mere error in reading the scale; e. g., (1) The faces of the prism may not be true planes, (2) the divided circle may not be uniform, (3) the center of the circular scale may not coincide with the center of the instrument, (4) the various adjustments may not be perfect, (5) there may be difficulty in fixing the position

of minimum deviation. These errors might be eliminated by repeating all the adjustments and observations many times and using different parts of the divided scale. There is no other way of allowing for them.

Questions.

1. Give both physical and mathematical definitions of the refractive index.
2. Why is monochromatic light used?
3. Why is the *minimum* deviation chosen?

XXXIV. MEASUREMENT OF RADIUS OF CURVATURE.

Glasebrook and Shaw, Practical Physics, pp. 339–343; *Edser, Light*, pp. 116–121; *Koklrausch*, pp. 174–176.

The *radius of curvature* of a surface may be determined from the size or position of the image which the spherical surface, regarded as a mirror, forms of a definite object. Method (A) below is especially applicable to the measurement of the radius of curvature of *convex surfaces*, and method (B) to *concave surfaces*.

(A) Two bright objects (see Fig. 36) are placed on a line at right angles to the axis of the spherical surface, the intersection of the line and the axis being at a considerable distance A, from the surface, and each object being at a distance $L/2$ from the axis. If the apparent distance between the images of the two objects be l, the radius of curvature of the surface is

$$r = \frac{2Al}{L \pm 2l},$$

the + sign being used for a concave surface and the − sign for a convex.

Proof.

(For convex mirror.)

Let d = true distance between the images, x = distance of images from mirror.

By geometry

$$\frac{L}{d} = \frac{A+r}{r-x}, \quad \frac{d}{l} = \frac{A+x}{A}, \quad \therefore \frac{L}{l} = \frac{(A+r)(A+x)}{(r-x)A}.$$

By the equation for spherical mirrors (see p 125).

(1) $$\frac{1}{x}=\frac{1}{A}+\frac{2}{r}$$

$$\therefore \frac{1}{x}-\frac{1}{r}=\frac{1}{A}+\frac{1}{r}$$

$$\therefore \frac{r-x}{rx}=\frac{A+r}{Ar}$$

$$\therefore \frac{L}{l}=\frac{A+x}{x}=1+\frac{A}{x}.$$

From (1) $$\frac{A}{x}=1+\frac{2A}{r}$$

$$\therefore \frac{L}{l}=2+\frac{2A}{r},$$

$$\therefore r=\frac{2Al}{L-2l}.$$

The radius of curvature should be found for both surfaces of a *double convex lens*. The lens, preferably the one used in Exp. XXXV, if that has already been performed, is fitted in a clamp in a darkened recess. At some distance are two vertical slits illuminated from behind by incandescent lamps (or the lamp filaments themselves may be used) and between them a telescope. The telescope and the lens are adjusted until, on looking through the telescope toward the lens, the illuminated slits are seen reflected from the near surface of the lens. Distinguish these images from the images produced by the rear surface of the lens by the change of focus necessary to make one pair of images most distinct, and then to make the other pair most distinct, or, by observing the two images of a light held just outside one of the slits. Remember that the telescope inverts. A paper scale is pinned over the lens so that the upper edge is just below the center of the lens. The telescope is focused upon the scale, and rotated until the vertical cross-hair bisects one of the images from the near surface of the lens, and the scale read where crossed by the cross-hair. (Esti-

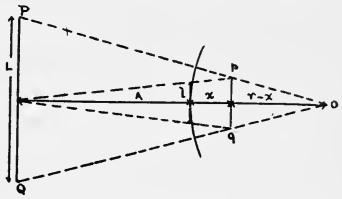

FIG. 36.

mate tenths of millimeters as always.) A similar reading is made for the other image. The difference between the two readings gives the apparent distance, l, between the images. At least six independent determinations of this distance should be made. Measure the distance, L, between the slits, the distance, A, from the lens surface to the line joining the slits and substitute in the formula.

From the two radii of curvature and the focal length, if known, calculate the refractive index, n, of the glass of the lens by means of the formula

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right), \text{ (see p. 125).}$$

(B) As a concave mirror we may use one of the surfaces of a *concave lens*, mounted in a lens-holder. To reduce reflection from the *other* surface, the latter may be covered by moist filter paper. The radius is determined from u, the distance of the object, v, the distance of the image and the formula

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{r}, \text{ (see p. 125).}$$

In locating the image, use is made of the fact that if the eye is a considerable distance off, a real image can be seen in space as well as a virtual image, and a wire, needle, or pointer is moved about until there is no parallax between it and the image; i. e., until, when the eye is moved about, there is no relative motion of the two.

A vertical wire illuminated by a lamp, behind which is a sheet of white paper, is a convenient object, and a second mounted wire is moved about until it coincides with the image of the first (see (B) Exp. XXXV). The image should be found for at least the following three typical positions of the object. For each position make several settings and from the means determine u and v, and from them determine r.

(1) Let the object be at a considerable distance from the mirror.

(2) Let the object be at the center of curvature of the mirror. In this position the image and the object coincide.

(3) Let the object be within the principal focus. For this position the wire locating the image must be on the other side of the lens. This wire is moved about until the prolongation above the lens of the image of the first wire coincides with what is seen of the second wire above the lens.

In the report, sketch the relative positions of mirror, image, and object, and state whether the image was magnified or diminished, erect or inverted.

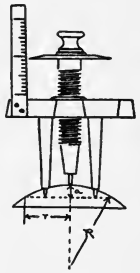

FIG. 37.

(C) If time permit, check your results with a spherometer (see p. 16). The spherometer should be read alternately on a plane surface and on the lens. Let a = difference in the two readings (see Fig. 37) and r = radius of the circle of the legs. Then the radius of curvature of the lens is

$$R = \frac{r^2 + a^2}{2a},$$

as may be easily shown.

Questions.

1. For what lenses would the first method of determining the radius of curvature be preferable, and when would the spherometer be preferable?
2. What objection is there to determining the radius of curvature of the farther face of a convex lens, considering it a concave surface?
3. What advantages has the method used in (B) for locating real images over the use of a screen?
4. How could you directly determine with a screen the center of curvature of a concave mirror?

XXXV. FOCAL LENGTH OF A LENS.

Text-book of Physics (*Duff*), pp. 392–398; *Watson's Physics*, pp. 471–479; *Ames' General Physics*, pp. 470–483; *Crew's General Physics*, pp. 466–469; *Edser, Light*, pp. 110–116; *Glazebrook and Shaw, Practical Physics*, pp. 343–352.

The focal length of a lens is the distance from the optical center of the lens to the focus for rays of light from an

infinite distance; i. e., for plane waves. If f is the focal length, u the distance of the object from the lens, and v that of the image, then, with the convention respecting signs given on page 125, for *all* lenses

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

(A) *Real Image.*—(If Exp. XXXIV has preceded, use the same lenses.) An "object," the lens, and a screen for receiving the image of the object, are mounted so that they can be moved along a graduated scale. A convenient form for the object is a wire cross or gauze, mounted in a black wooden support, and illuminated from behind by an incandescent lamp. The lens is clamped in a wooden frame movable along the scale. This should grasp the lens on the sides, leaving the top and bottom clear. The distance from the center of the lens to some point on the support must be determined once for all and applied as a correction to the readings. With object and screen in fixed positions that are recorded, the lens is adjusted until the image on the screen is as distinct as possible and its position is then recorded. This should be done several times, and the mean taken for the position of the lens. Keeping object and screen fixed and moving the lens about, another image will be found, for which similar observations should be made. Calculate f from the averages of all the values of u and v. The object and screen should then be shifted and the observations repeated. From the two sets of observations a mean value of f is deduced.

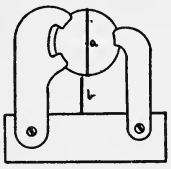

FIG. 38.

Study of Spherical Aberration.—Determine f for the central part of the lens by covering, with a pasteboard screen, all but a central disk of about one-third the diameter of the lens. Similarly determine f for the edge of the lens, using a diaphragm covering all but the edge.

Study of Chromatic Aberration.—Using the entire lens,

determine f for red light by placing red glass before the lens or object, and similarly for blue or green light.

In the report, tabulate, for comparison, the different mean values of f.

(B) *Virtual Image.*—In the preceding a real image was observed, but the focal length may also be found from observations of a virtual image. The following directions apply to a divergent lens. A vertical dark line on a white background serves as object. The image (between the lens and the object) is located with a short vertical wire, which is moved back and forth until a position is found where the image of the dark line seen through the lens (*a* in Fig. 38) appears at the same distance as the portion of the wire seen just below or above the lens (*b* in Fig. 38). This is secured when there is no relative motion of the image and this wire as the eye is moved horizontally, i. e., the wire appears as the prolongation of the image of the dark line or remains equidistant from such a prolongation. v will be the distance from the center of the lens to this wire which locates the image. Using a longer wire as the object and the dark line to locate the image, this method may be applied to the virtual image of a convergent lens.

Estimate the possible error of a typical measurement of f. Since practically all the error is in the location of the lens, the distance between the object and the screen may be considered free from error. If this distance is designated by w, the formula becomes

$$\frac{1}{u}+\frac{1}{w-u}=\frac{1}{f},$$

from which a formula may easily be derived for the possible error in f in terms of the possible error in u. The latter may be taken as the mean deviation from the mean in the location of the lens (see p. 4).

If Exp. XXXIV has preceded, determine the refractive index of the glass from the focal length and the radii of curvature.

Questions.

1. What is the minimum distance between object and screen to secure a real image? The maximum distance between object and lens to secure a virtual image?
2. What advantage is there in covering with a diaphragm all but the central portion of a lens? What disadvantage?
3. What is the cause of chromatic aberration?
4. What sort of a lens would show large spherical aberration? Large chromatic aberration?

XXXVI. LENS COMBINATIONS.

Edser, Light, Chaps. VI, VII, X; *Watson's Practical Physics*, pp. 358–367; *Drude's Optics*, pp. 44–46, 66–72; *Hastings' Light*, Appendix A. *Ames' General Physics*, pp. 488, 493, 494.

(A) *Determination of Principal Foci. Calculation from Focal Lengths and Separation.*—Let two lenses of focal lengths, f_1, and f_2, be separated a distance d. An object at a distance u from the first lens forms an image at a distance v determined by the equation

$$\frac{1}{v}=\frac{1}{f_1}-\frac{1}{u}=\frac{u-f_1}{uf_1}.$$

This image acts as an object for the second lens at a distance $d-v$. Hence the distance of the final image from the optical center of the second lens is given by the equation

$$\frac{1}{v'}=\frac{1}{f_2}-\frac{1}{d-v}=\frac{1}{f_2}-\frac{u-f_1}{du-df_1-uf_1}.$$

If u is infinite, v' is the distance, V, of the principal focus from the optical center of the second lens,

$$\therefore \frac{1}{V}=\frac{1}{f_2}-\frac{1}{d-f_1}; \text{ if } d \text{ is zero, } \frac{1}{V}=\frac{1}{f_1}+\frac{1}{f_2}.$$

Experimental Location.—Determine experimentally the position of this principal focus by finding the position in which an object must be placed for clear vision when it is viewed through the combination with a telescope focused for a very distant object. Compare with the calculated position. Determine similarly the other principal focus.

(B) *Determination of Focal Length.*—To determine the focal length, the position of the "principal points" must be known, as well as the principal foci. With thin lenses, the principal points practically coincide at the so-called "optical center." In thick lenses or lens combinations they may be considerably separated. The focal length is the distance from either principal focus to the nearer principal point. The equations for locating the image,

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f},$$

and for finding the linear magnification,

$$M=\frac{v}{u}$$

are applicable in all cases, if u and v are measured from the principal points.

Since it is difficult to locate the principal points, a method is often employed for determining the focal length which eliminates their position. Suppose that the linear magnification is M_1, when the distance of the object from the principal plane is u_1, and M_2 when the object is moved until its distance is u_2.

$$\frac{1}{M_1}=\frac{u_1}{v_1}=\frac{u_1}{f}-1, \quad \frac{1}{M_2}=\frac{u_2}{f}-1.$$

$$\therefore \frac{1}{M_1}-\frac{1}{M_2}=\frac{1}{f}(u_1-u_2). \qquad \therefore f=\frac{u_1-u_2}{M_2-M_1}M_1M_2.$$

If the focal length is small, the magnification should be determined with a micrometer microscope. A carefully graduated scale is a convenient object and the size of the image of one or more divisions is measured for two positions of the object a known distance apart. (Principle of Abbe's Focometer.)

Thus determine the focal length of the combination used in (A). Determine also the focal length of both the objective and the eye-piece of a telescope, using the same

one as employed in Exp. XXXVII, if that has preceded, and calculate the magnifying power for great distances.

In the report, draw careful figures of the lens combinations, representing the principal foci and principal points as calculated from the final mean results.

(C) If the principal points of a convergent system are close together, i. e., if the lens or lenses may be said to have an optical center, we may use the following approximate method: If w is the distance between object and image, and x that between the two positions of the lens for real images, $u=(w\pm x)/2$, $v=(w\pm x)/2$. Substituting these values in the formula we get

$$f=\frac{w^2-x^2}{4w}.$$

If time permit, try this method for the combination of lenses.

Describe a lens combination which (1) magnifies without distortion; (2) magnifies without chromatic aberration; (3) inverts without magnifying. (See references.)

XXXVII. MAGNIFYING POWER OF A TELESCOPE.

Glazebrook and Shaw, pp. 358–363; *Watson's Practical Physics*, pp. 367, 368.

The *magnifying power of a telescope* is the ratio of the angle subtended at the eye by the image as seen through the telescope to the angle subtended by the object viewed directly. (If Exps. XXXVI and XXXVIII have already been performed, use the telescope employed in those experiments.)

(A) *Direct Method.*—A minute mirror is attached to the telescope by wax so as to make an angle of about 45° with the axis and partly cover the aperture of the eye-piece. The telescope is focused upon a scale. A second scale is mounted parallel to the first and near the eye-piece, in such a

position that the observer's eye sees, side by side, the image of the scale viewed though the telescope and the image of the other scale reflected in the small mirror. From the ratio of the images of one or more scale divisions, and their distances, the angles are calculated, and from their ratio the magnifying power is deduced.

Find the magnifying power for at least six distances, making several observations for each. Also determine the *angular field of view* of the telescope by determining for each distance, r, the total distance on the scale, n, visible in the telescope. The angular field of view, in degrees, will be $57 \cdot 3\, n/r$.

The magnifying power defined above is very approximately equal to (1) the ratio of the magnitude of the image to the magnitude of the object when the two are in the same plane, and, for great distances, is equal to (2) the ratio of the focal length of the objective to the focal length of the eye-piece.

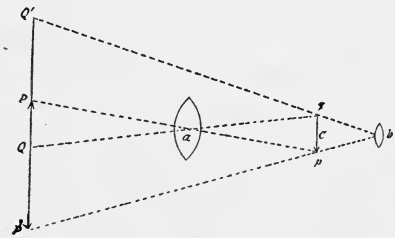

FIG. 39.

The eye-piece is of such short focus that the angle subtended by its image is practically the same as if the image were at infinity. For convenience we will consider the virtual image P' Q' (see Fig. 39) produced by the eye-piece to be at the same distance as the object $P\ Q$.

Since the telescope usually views objects at distances great compared with its own length, the angle subtended by the object viewed directly is practically $P\ a\ Q = p\ a\ q$, and that subtended by the image is $P'\ b\ Q' = p\ b\ q$. The ratio of these two angles, which may be taken as the ratio of the tangents, since the angles are small, $= a\ c \div c\ b =$ the ratio of the focal length of the objective to the focal length of the eye-piece; and also, since the length of the telescope is short compared with the distance of the object, this ratio $= P'\ Q' \div P\ Q$, or the ratio of the magnitude of the image to the magnitude of the object.

(B) The first approximate statement of the magnifying power furnishes another method for determining the magnifying power for different distances of the object. The telescope is directed toward a horizontal scale. The scale is viewed through the telescope with one eye and is also observed with the other eye by looking along the outside of the telescope. The eye-piece is moved in or out until the image appears at the same distance as the scale as viewed outside the telescope with the other eye, i. e., until there is no parallax between the scale and its image (no relative motion of the two as the eye is moved about). It may require some practice to secure this. Determine the number of divisions on the scale which, as viewed directly, are covered by the image of one or two large divisions as viewed through the telescope. If it is difficult to read the division on the scale viewed directly, two black strips may be moved along the scale until they include the image of one or more divisions as seen through the telescope, and the distance between these strips read off. Repeat the measurements of (A).

(C) The second method of defining the magnifying power of a telescope is useful in determining the *magnifying power for very distant objects*. Focus the telescope on some very distant object. Without changing the focus, remove the object glass and substitute for it a diaphragm with a rectangular opening. The ratio of the focal length of the objective to the focal length of the eye-piece is the ratio of a linear dimension of the aperture of the diaphragm, L, to the corresponding dimension, l, of the image of this aperture produced by the eye-piece.

Since the telescope was focused for parallel rays, the distance, u, of the object, L, from the eye-piece is numerically very nearly the sum of the focal lengths, $F+f$ (Fig. 40). $\therefore$ the distance of the image, l, formed by the eye-piece, is determined by

$$\frac{1}{v}=\frac{-1}{(F+f)}+\frac{1}{f}=\frac{-f+(F+f)}{f(F+f)}=\frac{F}{f(F+f)},$$

Hence,

$$\frac{L}{l}=\frac{u}{v}=\frac{F}{f}.$$

To measure l, a micrometer microscope (see p. 15) may be used, the microscope being in line with the axis of the telescope and focused upon the real image in space. L may

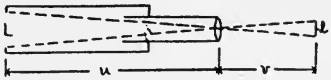

FIG. 40.

be measured with vernier calipers, or the same micrometer microscope may be placed opposite the other end of the telescope and L measured in the same way as l.

Questions.

1. Explain why the magnifying power should vary as you have found it to do with the distance of the object.
2. Which is preferable—to gain magnifying power by increasing the focal length of the objective, or by decreasing the focal length of the eye-piece? Why?

XXXVIII. RESOLVING POWER OF OPTICAL INSTRUMENTS.

Text-book of Physics (*Duff*), pp. 421, 422; *Watson's Practical Physics*, pp. 335–338; *Ames' General Physics*, pp. 483–487; *Mann, Advanced Optics*, pp. 11–18; *Drude's Optics*, pp. 235–236; *Hastings' Light*, pp. 70–72.

The magnification obtained with an optical instrument depends upon the focal lengths of its lenses, as has been seen in the case of the telescope. The ability to distinguish details of the image, i. e., the "resolving power," depends on the diameter of the aperture through which light enters the instrument.

If d is the distance between two details of an object at a distance D from an aperture whose width parallel to these details is a, they may be distinguished if

$$\frac{d}{D} \geqq \frac{l}{a}$$

where l is the wave-length of the light employed. If the aperture is circular, the equation is

$$\frac{d}{D} \gtrsim 1.2\frac{l}{a}$$

where a is the diameter of the aperture.

(A) *Resolving Power of Telescope.*—Metal gauze answers as a very satisfactory object for studying the resolving power, as it gives a great amount of uniform detail. Since this detail consists of rectangular lines, and the aperture of the object glass is circular, the determination of the maximum distance at which the lines are discernible will be more definite, if the aperture is made rectangular by placing a slit in front of the object glass.

Determine carefully by several settings, the maximum distance, D, at which the lines of the gauze, parallel to the slit, are perceived. The gauze should be illuminated from behind by monochromatic light (p. 124). Measure carefully the distance, d, between the centers of the adjacent wires of the gauze, and the width of the slit, a. Compare d/D with l/a; l may be obtained from Table XVIII. Repeat with other slits and other gauzes.

(B) *Resolving Power of Eye.*—With Porter's apparatus the resolving power of the eye may be determined for various apertures.

Four different gauzes, 37.6, 27, 20, and 14, meshes to the cm., respectively, may be viewed through four different apertures of diameters 1.00 mm., 0.65 mm., 0.53 mm., and 0.35 mm., respectively. The resolving power is detemined by finding the distance, D, from a slit of diameter a to a gauze, of which the distance between the centers of two adjacent wires is d, when the wires are separately discernible. From the mean position of several settings of a particular gauze for a particular aperture, d/D should be calculated and compared with $1.2\,l/a$. If ordinary light is used, l may be taken as 0.00006 cm. Use each aperture and gauze in succession.

Questions.

(1) Upon what does the illumination of the image of an optical instrument depend?

(2) When the diameter of the pupil of the eye is 4 mm., how far away may two points be distinguished which are 0. 2 mm. apart?

XXXIX. WAVE-LENGTH OF LIGHT BY DIFFRACTION GRATING.

Text-book of Physics (*Duff*), pp. 423, 437; *Watson's Physics*, pp. 529–532; *Ames' General Physics*, pp. 530–537; *Crew's General Physics*, pp. 488–491; *Edser, Light*, pp. 448–458; *Wood, Physical Optics*, pp. 168–180.

A *diffraction grating* consists of a great many lines ruled parallel and equidistant on a plane (or concave) surface. If the surface be that of glass, the grating is a transmission grating; if of metal, a reflection grating. If a transmission grating be placed perpendicular to homogeneous parallel light from a collimator (see Exp. XXXIII) and with the lines parallel to the slit, a series of spectra will be formed on either side of the beam of light transmitted without deviation. If n be the number or *order* of a particular spectrum counting from the center, θ the deviation or angle that the rays forming the spectrum make with the original direction of the light, a the grating space or average distance between the centers of adjacent lines, and l the wave-length of the light

$$l = \frac{1}{n} a \sin \theta.$$

The *deviation* θ may be observed by placing the grating on a spectrometer (see Exp. XXXIII where the adjustments of the spectrometer are described). The position of the telescope when in line with the collimator is read. The grating is adjusted parallel to the axis of rotation of telescope and collimator as one face of a prism is adjusted. For convenience in adjusting, the plane of the grating should be perpendicular to the line of two of the leveling-screws. This enables us to adjust the lines of the grating parallel

to the slit by means of one leveling-screw without altering the plane of the grating. The lines are parallel to the slit when the spectrum of some homogeneous light, e. g., from a sodium flame (p. 124), is as distinct as possible. When the plane of the grating is perpendicular to the incident light, the deviations (on opposite sides) of the two spectra of the same order should be equal. This adjustment is also secured when that part of the beam which is reflected back to the collimator appears co-axial with its object glass.

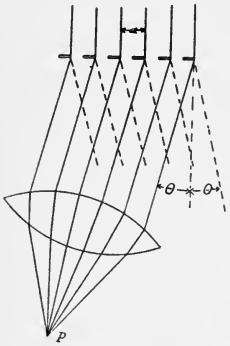

FIG. 41.

Determine first the wave-length of sodium light. For a final measurement of the deviation of any spectrum the mean of at least three measurements on each side should be taken. The deviations of all the spectra clearly visible should be obtained.

If the *grating space* be not too small it may be obtained by measurements on a dividing engine (p. 17), or with a micrometer microscope (p. 15). In determining the grating space with the dividing engine, secure the best possible illumination of the lines. Set the cross-hair of the microscope on a line and read the position of the divided head (circular scale). Watching the lines through the microscope, turn the screw, always in the same direction, until, for example, the tenth line is under the cross-hair, and read the circular scale. Then turn the screw until the tenth line from this is under the cross-hair, read the scale, and so on. Take ten such groups in different parts of the grating. Find the average grating space from the mean. When the grating space is very small, the wave-length of some well-known spectrum (e. g., sodium) is assumed in order that the grating space may be derived by reversing the process of finding the wave-length.

If time permit, determine the wave-length of as many other lights (colors) as possible (see p. 124).

Questions.

1. What do you observe as regards the width of spectra of different orders? What would this indicate as regards the dispersion if mixed light or light from an incandescent solid were used?
2. What is a normal spectrum and wherein does a prismatic spectrum differ from a diffraction spectrum? (See references.)

XL. INTERFEROMETER.

Text-book of Physics (*Duff*), p. 438; *Watson's Physics*, p. 540; *Mann*, *Advanced Optics*, Chap. V; *Wood*, *Optics*, Chap. VIII; *Michelson*, *Light Waves and Their Uses.*

The *interferometer* is an instrument for determining the number of wave-lengths of a monochromatic light contained in a given distance. For a description of the interferometer and the adjustments, see the references.

The interferometer will be used to determine the wave-length of sodium light, assuming a knowledge of the true pitch of the screw. This will illustrate the more practical and common, but also more difficult, utilization of the interferometer in determining a length, assuming a knowledge of the wave-length of the light employed.

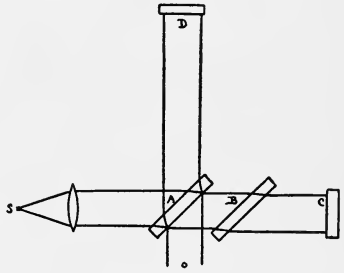

FIG. 42.

The light S (see figure) had best be monochromatic, e. g., a sodium flame. Initially, place the mirror D at approximately the same distance from the rear face of A as the distance from this surface to C. Adjust C until its image coincides with either of the images from D. (There will be two images owing to reflection from the two faces of A.) A slight adjustment will now give the fringes (alternate light and dark bands, preferably arcs of circles). The observer must look at A in a direction parallel to AD.

Move the mirror D by means of the worm, and count

the number of fringes which pass over the field of view. A needle in front of *A* may help as an index.

From the number of turns and fractional turns of the screw and the value of the pitch of the screw, find the distance *D* has moved and from this and the number of fringes which have passed, calculate the wave-length. Notice that the length of the path of the light changes by twice the displacement of the mirror *D*.

XLI. ROTATION OF PLANE OF POLARIZATION.

Text-book of Physics (*Duff*), pp. 476–479; *Watson's Physics*, pp. 580–582; *Ames' General Physics*, pp. 563–565; *Watson's Practical Physics*, pp. 370–377; *Edser, Light*, pp. 503–509; *Wood, Optics*, Chap. XIV; *Ewell, Physical Chemistry*, pp. 217–223.

Plane polarized light is obtained by passing light through a Nicol prism. If the light be then allowed to fall on a second Nicol prism that can be rotated, there will be two positions of this second prism in a complete rotation in which no light will pass through. If an optically active substance, such as a solution of cane sugar, be then introduced between the two Nicols, it will rotate the plane of polarization of the light which falls on the second prism, and then, to quench the light, the second prism must be rotated through an equal angle. Thus the rotation produced by the sugar is measured.

Monochromatic light must be used and a sodium flame is most convenient (see p. 124). The light rays must be made parallel before they fall on the polarizing prism, otherwise rays in different directions would pass through different thicknesses of the sugar and would consequently be rotated by different amounts. Parallel light may be obtained by putting the source at the principal focus of a convex lens through which the light has to pass before falling on the polarizing Nicol. The light must also be parallel to the axis of the Nicol.

The empty tube intended to contain the sugar solution is first placed in position between the prisms and the position of the analyzing Nicol noted, on the circular scale, when the light is quenched. This setting will be facilitated by using a screen to cover all but a small central part of the prism. It may be found that the Nicol can be rotated through an appreciable angle without the light reappearing. The best that can be done is to take the middle of this space as the position of extinction. The observation should be repeated a number of times and the mean taken. The analyzing Nicol should then be rotated through 180° and the zero reading in that position also noted. Sugar solutions of different strengths (which should be carefully made up and recorded) are then introduced in succession into the tube and the rotations they produce observed. The zero readings should be frequently repeated. The length of the tube should also be obtained, so that the rotation per decimeter may be deduced. With the results obtained, a curve should be plotted, rotations per decimeter being ordinates and concentrations abscissæ.

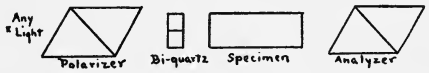

FIG. 43.

The apparatus here described is simple but very imperfect in its action. The sensitiveness is greatly increased by introducing between the polarizer and the specimen a so-called *biquartz*, two parallel, abutting, plates of quartz, one with left rotatory power and the other with right. A source of white light must be employed, for example, a frosted incandescent bulb, and the analyzer is set for equality of color in the two halves. There are two common colors, but the darker is preferable.

Fig. 44 explains the color changes. *R* and *L* are the two halves of the biquartz, viewed from the analyzer. The two halves are of such a thickness (3.75 mm.) that the plane of polarization of yellow

light is rotated through 90°. Owing to the rotatory dispersion the other colors will be rotated different amounts as shown by the letters *R* (red) and *B* (blue). If the analyzer is set to transmit light vibrations parallel to those which left the polarizer, the yellow light will be omitted and each half of the biquartz will appear of a purplish color ("tint of passage"). If the analyzer is displaced slightly clockwise, more of the red component on the right will be transmitted and less of the blue, and therefore this half will appear red and the other half will appear blue.

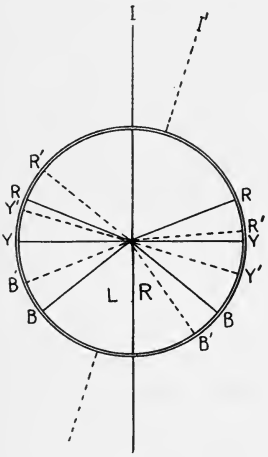

FIG. 44.

If a dextrorotatory specimen is placed between the biquartz and the analyzer, the directions of vibration of the different colors will be rotated to the positions indicated by the dotted lines and the analyzer must be rotated to a new position (*I'*), perpendicular to the emerging yellow vibration, in order to have the two halves the same color. With the help of the biquartz the analyzer can be set within about a tenth of a degree.

The effective thickness of the biquartz will not be correct (90 degrees rotation of sodium light), unless it is perpendicular to the axis of the Nicol prisms. This may be secured by using sodium light and analyzer set for extinction, and then placing the biquartz in such a position that there is still extinction when the analyzer is rotated 90°.

Repeat all the measurements with white light and the biquartz and plot the results on the same sheet with the preceding.

QUESTIONS.

1. How can the rotation be partially explained? (See references.)
2. What is the chemical characteristic of substances that are optically active in solution?
3. Wherein does the rotation produced by a solution differ from that produced by a magnetic field?
4. What would be the effect of using white light in the first part of the experiment?

29. Resistance-boxes.

A *resistance-box* consists of a number of resistance coils joined so that each one bridges the gap between two of a series of brass blocks placed in line on the cover of the box within which the coils are suspended. For each gap a plug or connector is also provided, and when the plug is inserted into the gap the resistance at the gap is "cut out" or practically reduced to zero. The coils are wound so as to be free from self-induction. The successive resistances are arranged in the same order, and are of the same relative magnitudes as the successive weights in a box of weights. By removing the proper plugs any combination of resistances can be obtained from the smallest to the sum of all. Before beginning work, it is advisable to clean the plugs with fine emery-cloth so that they may make good contacts, and thereafter care should be taken not to soil them with the fingers.

One important precaution in regard to the use of the resistance-box should be observed. If any of the plugs are in loosely, there will be some resistance at the contact. Hence, the plug should be screwed in firmly, but not violently. When any one plug has been withdrawn, the others should be tested before proceeding, for the removal of one may loosen the contact of the others. This precaution is especially important in making a final determination.

30. Forms of Wheatstone's Bridge.

The *practical measurement of a resistance* consists in comparing it with a known or standard resistance. Wheatstone's Bridge is an arrangement of conductors for facilitating this comparison, and consists essentially of six branches

which may be represented by the sides and diagonals of a parallelogram (see Fig. 45). The unknown resistance, R, and the known resistance, S, form two adjacent sides. The other two sides are formed by two conductors of resistance P and Q, which, however, do not need to be known separately, provided their ratio be known. One of the diagonals contains a battery and the other a galvanometer. If the ratio of P to Q is adjusted until no current flows through the galvanometer, $R:S::P:Q$. (See references under Exp. XLIV.)

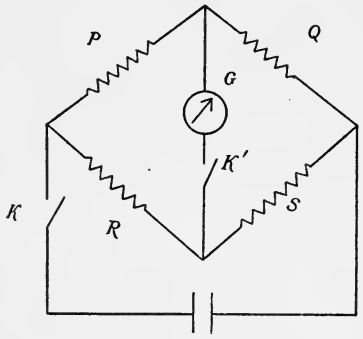

FIG. 45.

Two forms of the Wheatstone Bridge arrangement are in common use. One is called the *Wire (or meter) Bridge;* the other, which uses a box of adjusted resistances, is called a *Bridge Box.* In the wire bridge the "ratio arms" (whose resistances are P and Q) are the two parts of a uniform wire 1 meter long, and the ratio of P to Q is that of the lengths of the corresponding parts of the wire. The known resistance, S, may be that of a standard coil or one of the known resistances of a resistance-box.

The *Bridge Box,* or "Post-office Bridge," consists of a resistance-box with three series of resistances in line, forming three arms of the Wheatstone Bridge, the unknown resistance forming the fourth arm. The "ratio arms" consist of resistances of 1, 10, 100, 1000 (all of which are not always necessary), so that the calculation of the ratio is very simple. Keys for closing the battery and galvanometer branches are also usually mounted on the box.

31. Galvanometers.

Text-book of Physics (*Duff*), pp. 559-563; *Hadley's Electricity and Magnetism*, pp. 273-284; *Watson's Practical Physics*, §§170-174; *Ames and Bliss*, Appendix iii.

There are two chief types of reflecting galvanometers. In both the principle at basis is that if a magnet be placed in the plane of a coil of insulated wire, on passing a current through the coil both magnet and coil become subject to forces that tend to set them at right angles to each other. In the Thomson type the coil is fixed and the magnet suspended within the coil is free to turn, while in the d'Arsonval type the magnet, of a horseshoe form, is fixed, and the coil, suspended between the poles of the magnet, is free to turn.

The sensitiveness of the *Thomson galvanometer* is greatly increased in two ways: first, two magnetic needles, forming an astatic pair, are attached to the same axis of rotation, second, an external control magnet is used to weaken the restraint of the earth's magnetic force or even to overcome the earth's field and produce a suitable field of its own. The chief difficulty in greatly increasing the sensitiveness by means of the control magnet is that slight variations of the whole magnetic field, due to outside currents or movements of magnetic materials in or near the laboratory, disturb the needle.

On the *d'Arsonval galvanometer* variations of the external magnetic field have practically no effect, since its own magnetic field is very strong. On the other hand, the torsion of the fine suspending wire through which the current has to pass changes somewhat with the temperature, so that the zero reading of the galvanometer is subject to some change. The sensitiveness can be increased by increasing the strength of the magnet, but there is a limit to this, since small traces of iron are always present in the wire and insulation of the coil, and this, acted on by the magnetic field, exercises a magnetic control that is proportional to the square of the strength of the field. When

an extremely sensitive galvanometer for very accurate work is required, the Thomson type must be used.

A *ballistic galvanometer* is a reflection galvanometer of either type, so made that its period of swing is very long, so that it starts into motion only very slowly. If this condition be fulfilled, and if it be subject to only very slight damping of its motion, the galvanometer may be used for comparing quantities of electricity suddenly discharged through the coils of the galvanometer, for, practically speaking, all the electricity will have passed before the swinging system has appreciably moved from this position of rest. In these circumstances it can be shown that the quantity of electricity is proportional to the sine of half the angle of the first swing, or (since the angle is very small) practically to the deflection as read on the scale.

Two methods of reading the deflection of a galvanometer are in common use. In one, called the English or *objective* method, a beam of light reflected from the mirror of the galvanometer falls on a scale, forming a spot of light which moves as the needle or coil is deflected. In the other, called the German or *subjective* method, the image of a scale formed by the mirror of the galvanometer is read by a telescope with a cross-hair.

Devices for Bringing a Galvanometer to Rest.—For bringing to rest the needle of a ballistic Thomson galvanometer a coil is mounted on the outside of the galvanometer in front of the lower needle. The terminals of the coil are brought to a reversing switch by which the current from a cell can be sent through the coil in either direction. By suitably choosing the direction and duration of the current, the needles and mirror may be brought to rest. (A current in this coil affects the needle in the same manner as would a current in one of the regular galvanometer coils, but it is much more convenient to use a separate coil like this, which is readily accessible and which does not interfere with the other connections.)

The suspended system of either type of galvanometer

may also be brought to rest by short-circuiting the galvanometer by a simple key directly connected to the terminals. For, by Lenz's Law, the currents induced are such as to bring the moving coil or needle to rest. If the resistance of the coils is high, this method is slow, and the following more rapid method may be used. A coil in which a small bar magnet can be moved is placed in series with the short-circuiting key. By suitably moving the magnet in and out, currents are induced which will quickly bring the suspended system to rest.

32. Correction for Damping of a Ballistic Galvanometer.

Kohlrausch's Physical Measurements, §51; *Stewart and Gee's Practical Physics*, II, pp. 364–369.

In considering the throw proportional to the charge passing through the coils of a ballistic galvanometer, we assume that the galvanometer is free from damping; i. e., that the suspended system, needles, mirror, etc., experiences no resistance to turning. Since this is never realized, a correction must be applied to the throw.

The correction is not of importance where we compare throws, since the correction cancels out, but in much work with ballistic galvanometers this correction is very important.

Set the needle vibrating and record $n+1$ *successive* turning-points. From these we obtain by successive subtraction n successive full vibrations of the needle from one side to the other. Call the first full vibration a_1 and the last a_n. Then the correction by which each throw should be multiplied is $(1+\lambda/2)$ where

$$\lambda = \frac{\log a_1 - \log a_n}{n-1}$$

33. Galvanometer Shunts.

If a galvanometer of resistance G is shunted by a shunt of resistance S and if C is the whole current and C_1 the current through the galvanometer

$$\frac{C_1}{C}=\frac{S}{G+S}.$$

Galvanometers are frequently supplied with shunt-boxes in which the ratio of $S:G$ are 1/9, 1/99, 1/999, so that the values of $S:(G+S)$ are 1/10, 1/100, 1/1000. Such a shunt-box cannot easily be used with any galvanometer except that for which it was designed.

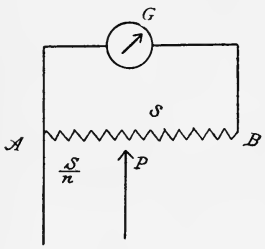

FIG. 46.

Universal shunt-boxes are now made which can be used with any galvanometer. Such a box consists of a series of high resistances connected as indicated in the figure. AB is a coil, of resistance S, connected to the galvanometer, of resistance G. Let the current through the galvanometer be C_1, and let the whole current be C. Then as above

$$C_1=\frac{CS}{G+S}.$$

Now let the battery circuit be connected to A and P, where the resistance of AP is S/n. Denoting the current through the galvanometer by C_1', and the whole current by C and making the proper changes in the above equation,

$$\frac{C_1'}{C}=\frac{S/n}{S/n+(S-S/n)+G}=\frac{1}{n}\frac{S}{S+G},$$

$$\therefore C_1'=\frac{1}{n}\frac{CS}{S+G}.$$

Hence when a current is connected to A and P, the galvanometer deflection is $1/n$ as great as when the same current is connected to A and B, or the sensitiveness is $1/n$ as great. By subdividing AB, the values of 3, 10, 100, etc., are given to n.

Shunting a Ballistic Galvanometer.—The formulæ stated above were deduced from Ohm's Law for steady direct currents. It can, however, be shown that shunts like the

above may be used in the same way with ballistic galvanometers through which *charges* of electricity are passed. To prove this, all we need to do is to show that charges, like steady direct currents, divide in a parallel arc into parts inversely as the ohmic resistances. Consider any one of several branches in a parallel arc. Let the part of the charge that passes through it be q, and let the magnitude of the instantaneous current through it, at time t after the beginning of the discharge, be i_1. The induced e. m. f. at that moment is $L_1 di_1/dt$ where L_1 is the self-inductance of the branch. Suppose the discharge is caused by connecting an e. m. f. E to the parallel arc for a short time and then disconnecting it, and let the whole time of rise and fall of the brief current be T. Then

$$q_1 = \int_0^T i_1 dt = \int_0^T \frac{E - L_1 \frac{di_1}{dt}}{R_1} dt,$$

$$= \frac{E}{R_1} \int_0^T dt - \frac{L_1}{R_1} \int_0^0 di_1,$$

$$= \frac{ET}{R_1}.$$

Hence the charges through the various branches are inversely as their ohmic resistances. If the above proof be carefully examined, it will be seen that it simply means that the total quantity due to the induced e. m. f. is zero, since the induced current in the first half of the process is opposite to that in the second half.

34. Standard Cells.

Text-book of Physics (*Duff*), pp. 584–585; *Watson's Physics*, pp. 806–807; *Watson's Practical Physics*, §§ 202–203; *Bureau of Standards Bulletin*, Nos. 67, 70, 71; *Henderson's Electricity and Magnetism*, pp. 176–182. *Ewell, Physical Chemistry*, pp. 334–336.

The *standard Daniell cell* consists of an amalgamated zinc rod dipping into a porous cup containing a solution of

sulphate of zinc, which, in turn, stands in a glass vessel containing a copper sulphate solution and a copper plate. To amalgamate the zinc rod, thoroughly clean it with sand-paper, dip it in dilute sulphuric acid, and rub over it a few drops of mercury with a cloth. The porous cup should be thoroughly cleaned inside and out. The copper plate should be cleaned bright with sand-paper. The porous cup is half-filled from a stock bottle with a solution of zinc sulphate (44.7 g. of crystals of c. p. zinc sulphate dissolved in 100 c.c. of distilled water). The zinc rod is introduced and the porous cup is placed in the glass vessel, which is filled, not quite up to the level of the zinc sulphate in the porous cup, with copper sulphate solution (39.4 g. of c. p. copper sulphate dissolved in 100 c.c. of distilled water). The copper plate is also placed in the outer vessel. After being set up, the cell should be short-circuited for 15 minutes and then allowed to stand on an open circuit for 5 minutes. The cell should not remain set up more than a few hours. When it is no longer needed, pour the copper sulphate solution back into the stock bottle and the zinc sulphate solution back into its bottle, unless the zinc has turned black, in which case throw the zinc sulphate away. The e. m. f. of the Daniell cell, prepared as above, is 1.105 international volts, correct to 0.2 per cent.

The *Clark cell*, which differs from the above in the fact that the copper is replaced by mercury and the copper sulphate by mercurous sulphate, is a more constant standard than the Daniell cell, but it needs to be treated with much greater care, since the passage of a very small current through it will alter the e. m. f. Hence it can be used only for null methods and kept in circuit for the briefest time possible. At temperature t its e. m. f. in volts is

$$1.433 - .0012(t - 15).$$

In the *cadmium cell* the zinc and zinc sulphate of the above are replaced by cadmium and cadmium sulphate. Its e. m. f. is

$$1.019 - .00004(t - 17).$$

35. Device for Getting a Small E. M. F.

In many experiments it is desirable to use an e. m. f. much smaller than that of a single cell. To get such an e. m. f., a box of very high resistance may be placed in series with a constant cell and any desired fraction of the whole e. m. f. may be obtained by tapping off from various

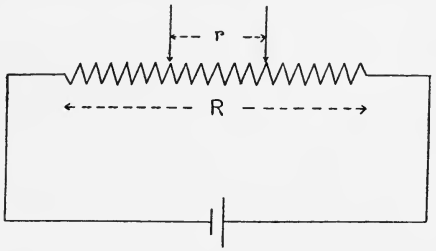

FIG. 47.

points; e. g., at the ends of a resistance r out of the total resistance R of the box (Fig. 47). The e. m. f. thus obtained may be found from Ohm's Law, but it must be noticed that the resistance between the terminals of r is the resistance of a parallel arc. If, however, the resistance of the branch circuit be proportionally *very large* and that of the cell proportionally *very small*, both may be omitted in the calculation.

36. Double Commutator.

It is sometimes desirable to be able to reverse two parts of a network repeatedly and at the same rate. For this purpose a double commutator is convenient. It consists of two two-part commutators mounted on a common shaft; *e. g.*, on opposite ends of the shaft of a small motor. If, for example, the battery used with a Wheatstone's Bridge be connected through one commutator while the galvanometer is connected through the other,

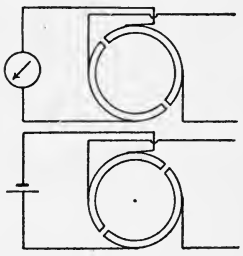

FIG. 48.

an alternating current will act in the arms of the bridge, while a direct current (or a succession of unidirectional pulses) will pass through the galvanometer.

37. Relation Between Electrical Units.

(E.S. = Electrostatic; E.M. = Electromagnetic.)

Ampere	$=10^{-1}$	C.G.S.–E.M. units of current.
Coulomb	$=10^{-1}$	C.G.S.–E.M. units of quantity.
Volt	$=10^{8}$	C.G.S.–E.M. units of electromotive force.
Ohm	$=10^{9}$	C.G.S.–E.M. units of resistance.
Farad	$=10^{-9}$	C.G.S.–E.M. units of capacity.
Microfarad	$=10^{-15}$	C.G.S.–E.M. units of capacity.
Henry	$=10^{9}$	C.G.S.–E.M. units of inductance.
Volt	$=\frac{1}{3}\times10^{-2}$	C.G.S.–E.S. units of electromotive force.
Coulomb	$=3\times10^{9}$	C.G.S.–E.S. units of quantity.
Microfarad	$=9\times10^{5}$	C.G.S.–E.S. units of capacity.

XLII. HORIZONTAL COMPONENT OF EARTH'S MAGNETIC FIELD.

Ames' General Physics, pp. 609–613; *Watson's Physics*, pp. 602–607; *Text-book of Physics (Duff)*, pp. 498–500; *Crew's General Physics*, pp. 319–323; *Hadley's Electricity and Magnetism*, pp. 92–98; *Watson's Practical Physics*, pp. 403–414; *Kohlrausch's Physical Measurements*, pp. 240–247; *Stewart and Gee's Practical Physics*, pp. 284–309.

In this experiment the horizontal component of the earth's magnetic field, at a point in the laboratory, is deduced from the period of vibration of a bar-magnet and the deflection of a magnetic needle produced by this same bar-magnet when placed at known distances E and W (magnetically) of the needle. The dimensions and mass of the magnet must also be obtained in order that its moment of inertia may be calculated.

If the period of vibration of the magnet be T in the place in which we wish to determine the horizontal component H, its magnetic moment be M, and its moment of inertia I, then

$$(1) \quad . \quad . \quad . \quad . \quad T = 2\pi\sqrt{\frac{I}{MH}}.$$

For, when the magnet is deflected through a small angle θ, the restoring couple is $MH \sin\theta = MH\theta$. Hence if the angular acceleration at that moment is α

$$-MH\theta = I\alpha$$

and

$$\alpha = -\frac{MH}{I}\theta$$

Since M, H, and I are constant, the motion is simple harmonic and T is given by (1).

If a magnetic needle at a distance d, E or W of this same bar-magnet, in line with it and the point where H is to be

determined, be deflected through an angle ϕ and, when at a distance d_1, be deflected through an angle ϕ_1

$$(2) \quad . \quad . \quad . \quad \frac{M}{H} = \frac{d^5 \tan \phi - d_1^5 \tan \phi_1}{2(d^2 - d_1^2)}.$$

Equation (2) is deduced from the expression for the force F produced by a magnet of magnetic moment M at a distance d in the direction of the axis of the magnet. For, if m is the strength of either pole of the magnet and $2l$ its magnetic length, the resultant force due to the two poles is

$$F = \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2} = \frac{4mld}{(d^2-l^2)^2} = \frac{2Md}{(d^2-l^2)^2}$$

By expanding the denominator we may also write this:

$$F = \frac{2M}{d^3}\left(1 + \frac{K}{d^2}\right),$$

in which K is approximately a constant. (If the length of the needle were also taken account of, this expression would remain unchanged, except that the value of the constant K would be different). If, under the force F and the component H of the earth's magnetic field, a magnetic needle makes an angle ϕ with the magnetic meridian, $F = H \tan \phi$. Hence,

$$\frac{M}{H}\left(1 + \frac{K}{d^2}\right) = \frac{d^3 \tan \phi}{2}$$

If, now, the distance be changed to d_1, and the deflection becomes ϕ_1 another equation similar to the above will be obtained and the elimination of K will give equation (2) above.

From equations (1) and (2) both H and M may be obtained when the other quantities have been measured.

(A) *To determine the period of vibration,* remove all movable iron (knives, keys, etc., included) to several meters from the vicinity of the entire experiment. Suspend the deflecting magnet, by means of a stirrup attached to a single strand of silk thread, in a box which has glass ends and sides and is surmounted by a glass tube through which the suspension passes. Level until the thread hangs in the axis of the tube. The magnet should be adjusted until it is horizontal as tested by comparison with a leveled rod attached to the outside of the box. Attach pointers to the opposite glass sides of the box (or adjust those provided) so that they are in line with the magnet at rest. Set the magnet vibrating through an angle not exceeding 10°.

Check any pendulum vibrations by judiciously pressing on the top of the glass tube. Then determine the period by the method of passages as in Exp. X. (see p. 54).

The magnet should be vibrated as near as is convenient to the place where the needle is deflected in the second part, i. e., where we wish to determine *H*.

(B) The instrument used in the *deflection part of the experiment* is called a *magnetometer*. It consists of a box with glass sides in which is suspended a mirror attached to either a small magnetic needle with a damping vane or a small bell magnet vibrating in a copper sphere. The sphere is placed at the center of a graduated bar upon which can be placed the deflecting magnet. Level until the suspending fiber is at the center of the bottom of the suspension-tube. If the needle or the damping vane does not swing free, a little additional leveling will be necessary.

A specially mounted large compass needle is used to adjust the magnetometer bar perpendicular to the magnetic meridian. By means of it a rod is placed in the direction of the magnetic meridian, and then, by means of a square, the magnetometer bar is made perpendicular to the rod. Place a telescope and scale about one meter from the magnetometer. See that the scale is perpendicular to the telescope. Adjust until the scale reflected from the mirror is clearly seen in the telescope (for directions for this adjustment see p. 25).

Place the magnet whose period of vibration has been determined on a small wood slide near one end of the magnetometer bar. Note the scale-reading on the magnetometer bar corresponding to the end of the magnet nearer the needle. When the needle comes to rest, record the scale-reading against the vertical cross-hair of the telescope. Remove the magnet several meters and read the zero. Replace the magnet at the same distance from the needle, but reversed, and again read the scale division corresponding to the vertical cross-hair. Make two similar readings with the magnet at an equal distance on the other side of

the needle. Read the zero before or after each reading and always estimate tenths of millimeters. Make four similar readings with the magnet at about two-thirds the distance on each side of the needle.

If the zero is somewhat unsteady, the following method will be found better. Omit zero readings and obtain the four deflection readings as rapidly as possible. Repeat this twice so that twelve readings in all are obtained. Take half the difference of each two successive readings as one value of the deflection. The final result will be the mean of all values so found. The extent to which they agree will indicate the reliability of the mean.

Measure the distance from the center of the scale beneath the telescope to the center of the suspension-tube of the magnetometer (i. e., the distance to the mirror). From this distance and the mean scale-reading for that distance, $\tan 2\phi$ is obtained (for it must be remembered that a reflected ray of light is turned through twice the angle that the reflecting mirror is turned through). Since ϕ is a small angle $\tan 2\phi = 2 \tan \phi$ very nearly. The distances from the needle to the near end of the magnet plus half the length of the magnet give d and d_1. At the *close* of the experiment, measure the length of the magnet with vernier calipers, and the diameter with micrometer calipers, and also weigh it. If l be the length, r the radius, and m the mass, the moment of inertia is:

$$I = m(\tfrac{1}{12} l^2 + \tfrac{1}{4} r^2).$$

In reporting, state the possible errors of the measurements of T, I, d, d_1, $\tan \phi$, $\tan \phi_1$.

Questions.

1. How could the true length of the deflecting magnet be obtained?
2. H having been obtained at one point in the room or building, what would be the easiest way of finding its value at any other point?
3. What are the other "elements" of the earth's magnetism?
4. If you have done Exp. XLIII, calculate the total force and the vertical component.

XLIII. MAGNETIC INCLINATION OR DIP.

Ames' General Physics, pp. 618–619; *Watson's Physics*, pp. 605–607; *Text-book of Physics (Duff)*, p. 506; *Crew's General Physics*, p. 312; *Hadley's Electricity and Magnetism*, pp. 99–102; *Watson's Practical Physics*, pp. 415–417; *Stewart and Gee's Practical Physics*, II, pp. 275–284.

(A) The dip, or inclination of the earth's magnetic lines of force to the horizontal, is found by means of a dipping needle or magnetic needle suspended on a horizontal axis which passes as nearly as possible through the center of gravity of the needle, with a vertical graduated circle for reading the angle of inclination. Such an apparatus is called a dip-circle, and includes a level and leveling screws for making the circle vertical, knife-edges for bearing the axis of the needle, a horizontal graduated circle for fixing the azimuth of the vertical circle, and an arrestment, with Y-shaped supports, for raising and lowering the needle and placing it so that its axis of rotation passes as nearly as possible through the center of the vertical circle.

The zero-line of the vertical circle must first be made vertical. This adjustment is made by means of the leveling screws and level just as a cathetometer is leveled (see p. 19). The circle must then be turned into the plane of the magnetic meridian. To attain this, advantage is taken of the fact that if the plane in which the needle is free to rotate be at right angles to the magnetic meridian, the needle must stand vertically; for in that position the horizontal component of the earth's magnetic force is parallel to the axis of rotation of the needle, and hence has no moment about that axis. The circle is, therefore, turned approximately east and west and then adjusted until the needle is vertical. This adjustment should be repeated several times, and each position should be carefully read with the assistance of a vernier if one is provided. A rotation of the circle through 90° from the mean position, as indicated by the horizontal circle, should then bring the plane of the circle to coincidence with the plane of the

magnetic meridian. By raising and lowering the arrestment, the needle is then placed on the knife-edges in the proper position for indicating the dip.

A single reading of the needle in this position would give a very imperfect value of the dip. Errors arise from various causes: (1) the axis may not roll freely on the knife-edges, owing to dust or friction. To remove any dust the axis and knife-edges should be brushed with a camel's-hair brush. The setting by means of the arrestment and the readings should be made at least twice, and both sets of readings recorded. (2) The axis of rotation of the needle may not be exactly at the center of the divided circle. This error may be eliminated by reading the position of both ends of the needle, one reading being from this cause as much too great as the other is too small. (3) The line of zeros on the vertical scale may not be truly vertical, and this would cause errors in the same direction in the readings of the ends of the needle. These errors may be eliminated by turning the vertical circle through 180° about a vertical axis and repeating the readings, for in these readings the quadrants on the other side of the zero line are used. (4) The axis of rotation may not pass exactly through the center of gravity of the needle. So far as the fault lies in the fact that the axis of rotation is to one side of the axis of figure of the needle, the error may be eliminated by reversing the needle in its bearings and repeating the readings; for in one position gravity will make the readings as much too great as in the other case it makes them too small. But gravity will also cause an error if the axis of rotation be in the axis of figure, but not at the center of the latter. The error will not be eliminated by reversing the needle on its bearings, but it will be if the magnetism of the needle is reversed and all of the preceding readings repeated; for then the other end of the needle will be lower and the error will be in the opposite direction. The reversal of the magnetism should be done under the direction of the instructor, the method of *double touch* being used.

In recording these various positions and readings, the side of the circle on which the scale is engraved may be called the face of the instrument, and similarly one side of the needle may be fixed upon as its face. Thus two readings of each end of the needle are to be made in each of the following positions:

(1) Face of instrument E, face of needle E;
(2) Face of instrument W, face of needle W;
(3) Face of instrument W, face of needle E;
(4) Face of instrument E, face of needle W.

The magnetism of the needle having been reversed, readings are to be again taken in the above positions. The final result is taken as the mean of these 32 readings.

(B) Another method of determining the dip is by means of an earth inductor in series with a ballistic galvanometer (p. 156). The earth inductor is first placed with the plane of its coils vertical and perpendicular to the magnetic meridian. It is then rotated through 180° and the throw d_1 noted. Several readings should be made. The plane of the coils is then placed horizontally and the throw d_2 on rotation through 180° noted. The ratio of d_2 to d_1 is the tangent of the dip.

Questions.

1. What other sources of error may there be in measurement by the dip-circle?
2. Would you be justified in making a calculation of "probable error" from the various readings with the dip-circle?
3. If you have performed Exp. XLII, calculate the total force and the vertical component.

XLIV. MEASUREMENT OF RESISTANCE BY WHEATSTONE'S BRIDGE.

Ames' General Physics, pp. 725–727; *Watson's Physics*, pp. 685–687; *Text-book of Physics* (*Duff*), p. 572; *Hadley's Electricity and Magnetism*, pp. 306–310; *Watson's Practical Physics*, pp. 432–437; *Kohlrausch's Physical Measurements*, p. 303.

The practical *measurement of a resistance* consists in comparing it with a known or standard resistance. For

resistances of medium magnitude, Wheatstone's Bridge is usually used (p. 153).

In joining the known and unknown resistances to the bridge, connectors should be used whose resistance is negligible; that is, less than the unavoidable error that may occur in determining the unknown resistance. In connecting the battery and galvanometer, no such precaution is necessary, for their resistances do not enter into the calculation. The galvanometer may be connected to either pair of opposite corners; but, where the greatest sensitiveness is required, if the galvanometer has a higher resistance than the battery, it should be in the branch that connects the junction of the highest two of the four resistances P, Q, R, S to the junction of the lowest two; while, if the battery has the greatest resistance, it should occupy that position. Two spring keys should be included in the connections, one in the battery arm and the other in the galvanometer arm. When testing for a balance, the battery key should be pressed first, then the galvanometer key. If taken in the reverse order, there might be a small deflection due to the self-induction of the various parts. These keys should be pressed for a moment only. Except for a final determination, it is not necessary to wait until the galvanometer has quite come to rest, for a lack of balance will be indicated by a sudden disturbance of the swing when the galvanometer key is pressed. The pressure of the galvanometer key should be brief, sufficient merely to indicate the direction of the initial movement.

In practice, it is best to use a box-resistance as nearly as possible equal to the unknown resistance. This comes to the same thing as saying that the box-resistance should be varied until a balance is attained when the parts of the meter wire are nearly equal. The reason for this preference is that the sensitiveness is then a maximum, or a slight lack of balance is most easily detected by the deflection of the galvanometer. The exact ratio of P to Q for a balance should be very carefully ascertained. At least six settings

should be made; and to secure independence of the settings, the eye should be kept on the galvanometer-scale and the reading of the bridge not examined until the setting has been decided on. The mean of these six is then taken. R and S should then be interchanged and six more settings made. This interchange will serve to eliminate the effect of lack of symmetry of the two sides of the wire bridge and its connections.

The structure of the galvanometer to be used, its coils, magnets, and connections, should be carefully examined and care taken that it is thoroughly understood (p. 155).

Three unknown resistances should be measured separately and then all in parallel. From the separate resistances the resistance of the conductors in parallel should be calculated and compared with the measurement of the same. The resistance of a wire should then be measured and its length and mean diameter obtained. From these data, the specific resistance of the material of the wire should be deduced. The temperature at which the resistance is measured should also be noted, and from the temperature coefficient of the material (Table XXII) the specific resistance at 0° C. calculated.

The possible errors of the measurements, and hence the extent to which the calculations should be carried, may be deduced from the mean deviation in each set of readings.

Questions.

1. Does the battery need to be a constant one?
2. What objections are there to allowing the battery circuit to remain closed?
3. Why is it difficult by this method to measure *very* large or *very* small resistances?

XLV. GALVANOMETER RESISTANCE BY SHUNT METHOD.

Kohlrausch's Physical Measurements, p. 325.

If a galvanometer of resistance G connected in series with a battery of resistance B and e. m. f. E and a box resistance R gives a deflection d and if C be the current

$$C = \frac{E}{R+B+G} = Kd$$

where K is a constant for the galvanometer. If now the galvanometer be shunted by a resistance S and the deflection be then d' and the current through the galvanometer C',

$$C' = \frac{E}{R+B+\frac{GS}{G+S}} \times \frac{S}{G+S} = Kd'.$$

Hence

$$\frac{(R+B)\ (G+S)+GS}{S(R+B+G)} = \frac{d}{d'}$$

and from this G is readily deduced provided B is known. Usually a battery of such low resistance can be used that B is negligible compared with R and may be omitted; otherwise B must be obtained as in Exp. LIII. The galvanometer should be connected through a commutator and several readings on both sides should be made.

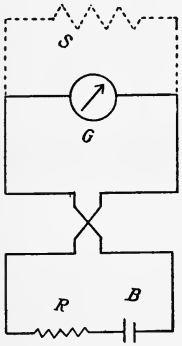

FIG. 49.

If the e. m. f. of the cell supplied is too great, a suitable fraction of it should be employed (p. 161).

As a check, the determination of G should be repeated, a different value for S being used. If the galvanometer is *very* sensitive, its resistance must be found from two readings with shunts. A suitable formula is readily worked out.

If R should be very great compared with the other resistances, the formula may be simplified. This will usually be the case if the galvanometer is very sensitive or of low resistance. The quantities added to R in the first two equations may then be neglected and we get

$$\frac{G+S}{S} = \frac{d}{d'}.$$

XLVI. GALVANOMETER RESISTANCE BY THOMSON'S METHOD.

Hadley's Electricity and Magnetism, p. 321; *Kohlrausch's Physica Measurements*, p. 328; *Stewart and Gee's Practical Physics*, II, p. 140–142.

The *resistance of the coils of a galvanometer* may be found by means of Wheatstone's Bridge as the resistance of any ordinary conductor is found. This would require the use of a second galvanometer. The second galvanometer, for detecting when the bridge is balanced, is frequently unnecessary. The condition for a balance is that, when the branch in which the galvanometer is usually placed is closed by a key, no current shall flow through it. If a current did flow through it, a change would take place in the currents in the other arms. Now the presence of a galvanometer in one of these other arms enables us to test whether any change in the distribution of the currents takes place on the key's being pressed. Hence, in Thomson's method for galvanometer resistance the galvanometer is placed in the "unknown" arm and a spring key, K, is placed in the branch in which, in the ordinary arrangement of Wheatstone's Bridge, a galvanometer is found. A diagram to illustrate the connections is given in figure 50.

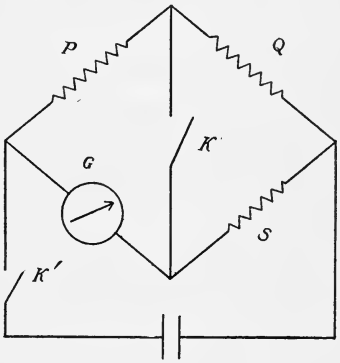

FIG. 50.

From the above it will be seen that in this method a balance is obtained when *the deflection of the galvanometer does not change* on the key, K, being pressed. Two practical difficulties are met with. The first is that the deflection of the galvanometer before the key is pressed may be so large that it cannot be read. When the galvanometer is of

the Thomson type (p. 155), this difficulty may be overcome by turning the control magnet until the deflection can be read (the zero position of the galvanometer could, of course, not then be read on the scale, but that is not necessary). In the d'Arsonval type of galvanometer there is no such way of overcoming this difficulty, and so this method is not so easily applied to such a galvanometer. The second difficulty is that if the battery be a variable one, the galvanometer will not give a steady deflection. Hence, a constant battery of the Daniell or Gravity type should be used (p. 159). It may also be necessary to decrease the current through the bridge and galvanometer by putting considerable resistance in series with the battery, or a fraction of the e. m. f. of the cell may be used (p. 161).

In the experiment it is better to use a bridge-box instead of a wire bridge, for the condition for sensitiveness, that the arms should be as nearly equal as possible, still holds, and the resistance of a wire bridge is usually very small compared with that of the galvanometer. Beginners sometimes find difficulty in deciding on the proper connections. The best way is to consider what the connections would be in the ordinary use of Wheatstone's Bridge, and then consider the modifications introduced in the present method. If possible, ratio arms of 1000 to 1000, 100 to 1000, and 10 to 1000 should be used in succession to obtain successive approximations. The last should give the resistance to two places of decimals (if one ohm is the least box-resistance), but the decreasing sensitiveness may prevent the latter ratios from giving more accurate results than the first.

If the galvanometer has more than one coil, the resistance of each should be measured separately and then the resistance of all in series. This will afford a check on the work.

Question.

1. Describe carefully the swinging system, coils, control magnet, and connections of the galvanometer used.

XLVII. MEASUREMENT OF HIGH RESISTANCES (1).

Watson's Practical Physics, pp. 460–461; *Henderson's Electricity and Magnetism*, pp. 66–72.

The method of Wheatstone's Bridge is not suitable for measuring very high resistances. One method is to connect the unknown resistance X, a battery of negligible resistance and e. m. f. E, and a sensitive galvanometer of resistance G in series. If the current be C,

$$C = \frac{E}{X+G}, \text{ giving a deflection } d.$$

Now replace X by a known resistance, R, and shunt the galvanometer by such a resistance, S, that the deflection is readable. By considering the total current and the part C' of the total current that passes through the galvanometer, we readily find that

$$C' = \frac{E}{R+\frac{GS}{G+S}} \frac{S}{G+S}, \text{ giving a deflection } d'.$$

Hence,

$$\frac{R(G+S)+GS}{S(X+G)} = \frac{d}{d'},$$

and from this X is readily deduced. G may be found as in Exp. XLV or XLVI; but if $(G+S)/S$ is known and G is small compared with X and R, the resistance of the galvanometer need not be determined. Many galvanometers are provided with shunt boxes, for which $S/(G+S)$ is 0.1, 0.01, or 0.001.

The galvanometer should be connected through a commutator, and several readings on both sides should be made to obtain a reliable mean.

As a check, repeat the measurements with a different value for R and a different value for S.

XLVIII. MEASUREMENT OF HIGH RESISTANCES (2).

Text-book of Physics (Duff), pp. 537–538; *Watson's Physics*, pp. 656–657; *Ames' General Physics*, pp. 658–659; *Henderson's Electricity and Magnetism*, pp. 71–75; *Hadley's Electricity and Magnetism*, pp. 202–210; *Watson's Practical Physics*, pp. 569–571.

A very high resistance, such as the insulation resistance of a cable or the resistance of cloth, paper, wood, etc., may be measured by finding the rate at which the electricity in a charged condenser leaks through the conductor. An electrometer is used to find the change of potential of the condenser and from this the rate of loss of its charge is deduced. The Dolezalek form of Kelvin's quadrant electrom-

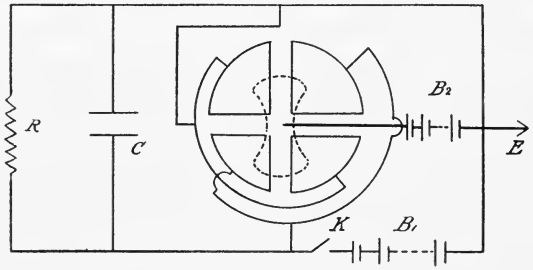

FIG. 51.

eter is suitable. Its needle is kept charged to a high potential by being connected to one pole of a battery of small cells, the other pole being grounded.

To find the insulation resistance of a cable the whole of the cable except the ends is immersed in a tank of salt water which is connected to the earth. One of the ends is carefully paraffined to prevent surface leakage and the core of the other end is connected to the insulated pair of quadrants. If the cable is sheathed with metal, immersion is not necessary. Other materials, such as those mentioned, are pressed between sheets of tinfoil, one sheet being connected to the earthed quadrants and the other to the insulated quadrants.

Let V_1 = potential given the condenser on closing the key K. The charge Q in the condenser and cable $= C\ V_1$ where C is their joint capacity. Upon opening K the charge flows through a resistance R_1 for a time, t, R_1 being the insulation resistance of the cable, the condenser, and the electrometer and keys in parallel.

Since the current at the time t equals V/R_1 by Ohm's law, and also equals the rate of decrease of Q or of CV

$$\frac{V}{R_1} = -C\frac{dV}{dt}$$

$$-C\frac{dV}{V} = \frac{dt}{R_1}$$

Integrating between the limits $t = o$ when $V = V_1$ and $t = t$ when $V = V_2$ we get

$$C \log_e \frac{V_1}{V_2} = \frac{t}{R_1}$$

$$\therefore R_1 = \frac{t}{C \log_e \frac{V_1}{V_2}} = \frac{0.434\ t}{C \log \frac{d_1}{d_2}}$$

where d_1 and d_2 are the initial and final deflections of the electrometer from the zero position.

The zero should be determined both before V_1 is found and after V_2 is found. As it is very apt to vary slightly, more reliable results can be attained by continuing to read V at intervals (e. g., every half-minute) until it has fallen to about one-half of its original value. From a curve drawn to represent V and t, two reliable points may be chosen to give values for V_1 and V_2 to be used in the calculation.

A subdivided condenser is desirable in order that a capacity giving a sufficiently rapid fall of potential may be chosen.

The total insulation resistance, R_2, of the other parts in parallel with the cable are found by disconnecting the cable and making a second set of observations as above.

The insulation resistance, R, of the cable is then deducible for

$$\frac{1}{R_1}=\frac{1}{R}+\frac{1}{R_2}.$$

The capacity, C_1, of the cable can be compared with that of the condenser, C_2, by the method of "divided charge." First charge the condenser and observe its potential by the electrometer and let the deflection be d_1. Then connect in the cable and let d_2 be the new deflection. Since the total charge Q remains unchanged,

$$Q=C_2V_1=(C_1+C_2)V_2$$

and, since the deflections are proportional to the potentials,

$$C=C_1+C_2=\frac{d_1}{d_2}C_2.$$

Questions.

1. Why should one pole of the battery that charges the needle be grounded?
2. Why must keys of specially high insulation be used in this method?
3. Calculate the capacity of the cable in electrostatic units from rough measurements of its dimensions and reduce to microfarads (see p. 162).

XLIX. MEASUREMENT OF LOW RESISTANCES (1).

Very low resistances cannot be measured by the Wheatstone Bridge method, because the unknown resistances of the connections are not small compared with the resistance to be measured. The simplest method for low resistances is a "fall of potential" method. A current is passed through the resistance, the current is measured by an ammeter and the difference of potential is measured by a voltmeter; then the resistance is known from Ohm's Law. For very low resistances the fall of potential will be very small and an instrument much more sensitive than any commercial voltmeter must be used. Instead of a voltmeter a sensitive galvanometer of *high* resistance, or a low resistance galvanometer *in series* with a high resistance, is used and the

value of a scale division of the galvanometer regarded as a voltmeter is found by a separate experiment.

Let the resistance to be measured be x, and let the difference of potential at its ends when current C passes through it be e. Then

$$x = \frac{e}{C}.$$

C, which should be large, may be measured by an ammeter. To find e we must know the constant, K, of the galvanometer considered as a voltmeter; that is, the number of volts per unit deflection. If the deflection is D

$$e = K.D$$

To find K apply to the galvanometer a small fraction of the e. m. f., E, of a Daniell's cell (p. 159). For this purpose connect the cell in series with a very high resistance box and a box of moderate resistances and join the galvanometer to the ends of one of the small resistances, r, choosing r so that the deflection, d, will not be very different from D. Then if the resistance of the galvanometer be great compared with r (see p. 161) and if the total resistance in series with the battery be R, the e. m. f. acting on the galvanometer is Er/R.

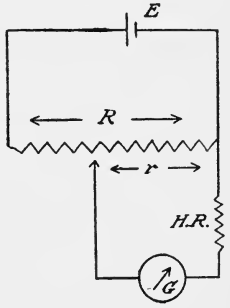

FIG. 52.

Hence

$$K = \frac{r}{R}\frac{E}{d}.$$

As a check on the work redetermine K using a different value of r. From the above equations x is found.

In the first part of the experiment place a commutator in the main circuit so that C may be reversed and the effect of thermo-electric forces at the contacts eliminated, and connect the galvanometer through a second commutator so that lack of symmetry in its deflection may be eliminated.

Thus D will be the mean of four readings. Exactly similar precautions should be observed in the second part. Close the currents only for the shortest possible times necessary to make the readings, otherwise heating may occur and resistances (especially the unknown x) may change.

The determination should be repeated several times with different values of C. If the work has been reliable, D should be proportional to C. Note also the temperature of the specimen and calculate its resistivity from its resistance and dimensions.

Questions.

1. If r had not been negligible compared with the galvanometer resistance how would this have appeared in the course of the work?
2. Find the equation that must replace the above if the resistance of the battery is not negligible compared with R and if r is not negligible compared with the galvanometer resistance.

L. MEASUREMENT OF LOW RESISTANCES (2).

Henderson's Electricity and Magnetism, pp. 57–58; *Stewart and Gee's Practical Physics*, II, pp. 177–181.

When a *standard low resistance* (0.01 or 0.001 ohm) is available, a conductor of low resistance x may be connected in series with it and a battery, and a very sensitive voltmeter, or a high-resistance galvanometer, serving as a voltmeter, may be used to compare the falls of potential in x and the standard. The resistances will be proportional to the falls of potential.

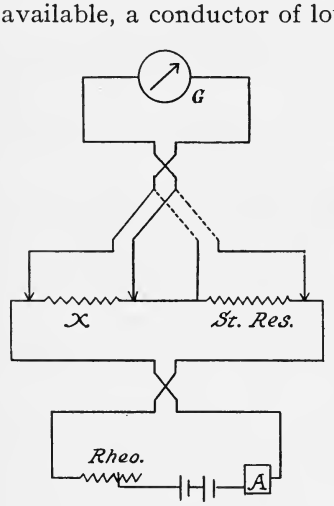

FIG. 53.

Connection with the battery should be made through a commutator to reverse thermal effects at the connections, and the galvanometer should be connected through a second commutator to eliminate asym-

metry of the galvanometer readings. Thus each final reading will be the mean of four separate readings.

The currents should be closed for the shortest times sufficient for the readings, to avoid heating. Note the temperature of the specimen.

Questions.

1. What are the comparative advantages and disadvantages of this and the preceding method?
2. Why is a high-resistance galvanometer to be preferred?
3. Will poor contact have as much effect as in a measurement of low resistance by Wheatstone's Bridge? Why?

LI. MEASUREMENT OF LOW RESISTANCES BY THE THOMSON DOUBLE BRIDGE.

Watson's Practical Physics, pp. 465–469; *Stewart and Gee*, II, pp. 182–187.

In *Thomson's Double Bridge* the errors of the contacts in the use of Wheatstone's Bridge are avoided. Its principle is, in fact, that of the fall of potential method (Exp. L) the direct comparison of the falls of potential being replaced by a null method. This method is applicable to extremely low resistances as well as to medium resistances.

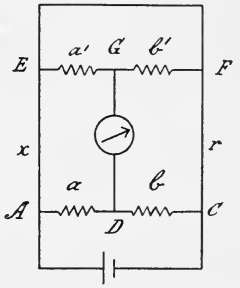

FIG. 54.

In the diagram x is the resistance to be measured and r a standard known resistance; a may be made 10 or 100; and b may be made 100, 1,000, 10,000. Similarly, a' may be made 10 or 100 and b' 100, 1,000, 10,000. Now it can be shown that if there is no current in the galvanometer, and if the ratios a/b and a'/b' are equal,

$$\frac{x}{r}=\frac{a}{b}=\frac{a'}{b'};$$

hence a value of r (which is variable) and values of a, b, a' b', are sought which give no current in the galvanometer, and from these x is calculated.

The form of Double Bridge made by Hartmann and Braun is very satisfactory. The correspondence of parts to parts of the diagram is readily traced. The ratio of a to b can be varied from 100 to 100 to 10 to 10,000; moreover, a and b may be interchanged and so the ratio reversed. Similar remarks apply to a' and b'. Thus values of x/r varying from 10/10000 to 10000/10 may be measured. The variable r may be varied from 0.044 down to 0, but can hardly be read with an accuracy of 1% below 0.001. Hence values of x between 0.001/1000 or 0.000001 and 0.044×1000 or 44, may be measured by the bridge.

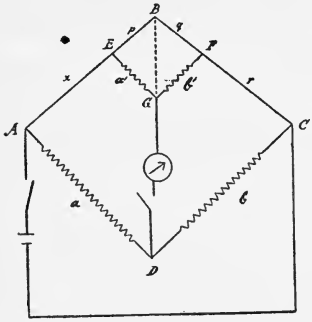

FIG. 55.

Care must be taken not to injure the standardized bar by scraping the contact maker along it. The contact maker must be raised for each movement. Do not allow the sharp jaws of the clamps to come down on the bar too suddenly, for they might cut into the bar somewhat.

Test as many as possible of the following materials:
(1) Brass. (2) Iron. (3) Copper. (4) Zinc. (5) Lead. (6) Carbon. (7) Rail Bond,
and calculate the Specific Resistance of each.

Proof of Formula.

Regard Thomson's Double Bridge as a modified Wheatstone's Bridge, the modification consisting in the paralleling of parts of the arms AB and BC as indicated. Let G and D be points at the same potential as indicated by the galvanometer. $E\ B\ F$ is the heavy conductor joining the unknown and the standard. Let B be a point in it at the same potential as G. We may suppose B and G permanently connected. Let the resistance of p and a' in parallel be m, and that of q and b' be n. Then, by the Wheatstone Bridge formula:

$$\frac{a}{b} = \frac{x+m}{r+n}.$$

Now if we show that

$$\frac{a}{b}=\frac{m}{n},$$

it will follow that

$$\frac{a}{b}=\frac{x}{r}.$$

To show this, note that

$$\frac{a}{b}=\frac{a'}{b'}=\frac{p}{q}$$

$$\therefore \frac{a'}{a'+p}=\frac{b'}{b'+q}.$$

Now

$$m=\frac{a'p}{a'+p},\quad n=\frac{b'q}{b'+q}$$

$$\therefore \frac{m}{n}=\frac{p}{q}=\frac{a}{b}.$$

Hence

$$\frac{x}{r}=\frac{a}{b}.$$

Questions.

1. Considering this as a modified fall of potential method, why should a, b, a', b', be of very large and $E\ B\ F$ of very small resistance?
2. Does the battery current need to be steady? Why?
3. Could an alternating current be used in any circumstances?

LII. COMPARISON OF RESISTANCES BY THE CAREY-FOSTER METHOD.

Watson's Practical Physics, pp. 442–446; *Schuster and Lees' Practical Physics*, pp. 307–309; *Henderson's Electricity and Magnetism*, pp. 53–57; *Stewart and Gee's Practical Physics*, II, pp. 158–170.

To find very accurately the difference between two very nearly equal resistances R and S, connect them and two other nearly equal resistances, P and Q, as indicated in the diagram, where ab is a very uniform wire, which we shall suppose to have a resistance of more than 1 ohm. Let the resistance of unit length of the wire ab be p. Let the distance ad be x_1, when a balance has been obtained in the usual way. Then exchange R and S, and again obtain a balance. Denote the new value of ad by x_2. Since P and Q

have not been changed and the total resistance R, S, and ab was not changed, it is clear that $R+x_1p=S+x_2p$, or $R-S=(x_2-x_1)p$. To find the value of p, replace R by a standard 1-ohm coil, and S by a heavy connector of negligible resistance, and proceed as above; then $p(x_2-x_1)=1$.

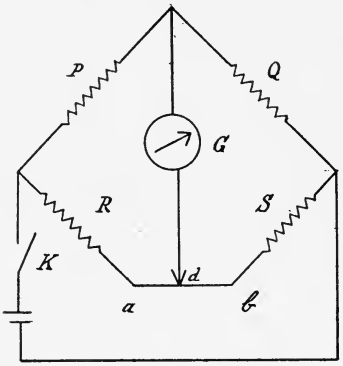

FIG. 56.

The exchange of R and S is made by means of a special key designed so that the resistance of the connections will remain the same (see Fig. 57).

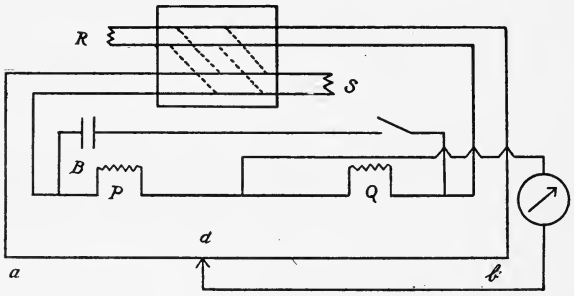

FIG. 57.

To compare a box of unknown errors with a standardized box, the difference between each resistance of the former and a corresponding resistance of the latter is found by the above method.

To calibrate two boxes, put one in place of R and replace S by a standard 1-ohm coil and so find exactly the value of each 1-ohm unit in the box. Then replace the standard by the other box, in position S, and compare the 1-ohm units of the second box with those of the first box. Then compare a 2-ohm unit in one box with two 1-ohms in the other, and so on. Special care must be taken to avoid confusion in making the calculations, and for this purpose the box resistances may be denoted by I_1, I_2, II_1, II_2, etc., for one box, and I'_1, I'_2, II'_2, etc., for the other.

Questions.

1. State the formula for Wheatstone's Bridge before and after R and S are interchanged and therefrom deduce the above formula.
2. Do P and Q need to be exactly equal, and why?

LIII. BATTERY RESISTANCE BY MANCE'S METHOD.

Hadley's Electricity and Magnetism, p. 322; *Watson's Practical Physics*, p. 475; *Schuster and Lee's Practical Physics*, pp. 303–306.

The *resistance of a battery* may be determined by placing it in the "unknown arm" R of a Wheatstone's Bridge (p. 153). In this case there will be a current through the galvanometer when the bridge battery is not connected. But if P, Q and S be adjusted until there is no change in the deflection when the key of the bridge battery is pressed, the points to which the galvanometer is connected will be at the same potential so far as the effect of the bridge battery is concerned. Since, when the adjustments are right, the bridge battery sends no current through the galvanometer, this battery may be removed and the key alone will serve to test the adjustment of P, Q, and S.

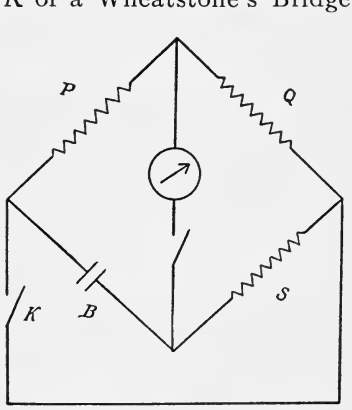

FIG. 58.

If the deflection of the galvanometer is too great to be readable, the control magnet (in the case of a Kelvin galvanometer) may be used to bring the needle back, or the galvanometer may be shunted or a resistance put in series with it. Most cells vary slightly in resistance and e. m. f. when on closed circuit; hence, the keys should not be pressed longer than is necessary.

In Lodge's modification of Mance's Method a condenser is placed in series with the galvanometer. There will then be no continuous current through the galvanometer; but, if the adjustments of P, Q, and S are not right, on pressing the key by which the adjustment is tested the galvanometer will be momentarily deflected.

Questions.

1. Why is there a slow movement of the galvanometer needle when the keys are kept pressed?
2. Should the condenser be of large or small capacity? Would a Leyden jar do?

LIV. TEMPERATURE COEFFICIENT OF RESISTANCE.

Text-book of Physics (*Duff*), p. 564–566; *Ames' General Physics*, pp. 731–732; *Watson's Physics*, pp. 681–682; *Hadley's Electricity and Magnetism*, p. 294; *Henderson's Electricity and Magnetism*, pp. 95–101.

The resistance of most solids increases as the temperature rises; carbon is one of the exceptions, for its resistance decreases. For moderate ranges of temperature the resistance is approximately a linear function of the temperature or, if R_0 be the resistance at 0° and R that at $t°$,

$$R = R_0(1 + at)$$

The constant a is called the temperature coefficient of the material. It may be defined as the change per ohm, referred to the resistance at 0°, per degree change of temperature.

The change of resistance can be most conveniently studied by the box form of Wheatstone's Bridge (p. 154).

(A) For finding the *temperature coefficient of a wire* such as copper, a length sufficient to give several ohms resistance

should be used. The determination of the temperature coefficient does not require that the dimensions of the specimen should be known, but the specific resistance of the specimen might as well be determined at the same time. Hence the length and mean diameter of the wire should be carefully measured. The wire should then be soldered to heavier lead wires and immersed in a bath of oil, and its resistance determined at intervals of about 10° as the temperature is raised. The thermometer should be placed inside the coil so as to be as nearly as possible at the temperature of the latter. It will be an improvement if the coil and thermometer are in a test-tube that is immersed in the bath, the opening of the tube being closed with cotton-wool.

To keep the temperature constant, while measuring the resistance, would be difficult. The following method will be found to give much better results: Having measured the resistance at the temperature of the room, adjust the known resistance of the bridge so that there would be a balance if the resistance of the wire were increased 4 or 5 per cent. The galvanometer will be deflected. Now heat the wire *very slowly* and the galvanometer reading will begin to drift toward zero. When it just reaches zero, read the thermometer and continue the process step by step.

The various resistances and temperatures should then be plotted in a curve that should be approximately a straight line. If exactly a straight line is obtained, the temperature coefficient should be calculated from two reliable and widely separated points on the curve. Let R and R' be the resistances at t and t', respectively. Substituting these values in the above equation we shall get two equations from which R_0 can be eliminated.

If the plotted readings give a distinct curve, the resistance must be expressed as a quadratic function of the temperature.

$$R = R_0(1 + at + bt^2)$$

From three points on the curve three equations may be written down and from these a and b may be calculated.

(B) For finding the *temperature coefficient of carbon* an incandescent lamp may be used. As it would be difficult to determine accurately the temperature of the filament in the exhausted bulb by the preceding method, water may be used for the bath and two careful determinations of the resistance made, the first being while the water is at about the temperature of the room, and the other when the water is boiling. In each case the final determination of the resistance should not be made until the temperature of the filament has become constant, as is indicated by its resistance becoming quite constant. The leads, where they are immersed in the water, should be carefully insulated with tape.

LV. SPECIFIC RESISTANCE OF AN ELECTROLYTE.

Watson's Practical Physics, pp. 475–486; *Henderson's Electricity and Magnetism*, pp. 80–84; *Ewell's Physical Chemistry*, pp. 54–57; *Kohlrausch's Physical Measurements*, pp. 316–321.

The object of this experiment is to determine the *specific resistance of an electrolyte*—for instance, solutions of copper sulphate of different concentrations. The box form of Wheatstone's Bridge is most suitable for the purpose (p. 154).

A steady current from a battery and a galvanometer to determine when there is a balance, as ordinarily used with Wheatstone's Bridge, cannot satisfactorily be used in measuring the resistance of an electrolyte, for a steady current produces in a short time polarization at the electrodes. This polarization leads to too high an estimate of the resistance of the electrolyte, for when no current flows through the galvanometer, the three other arms of the bridge are balancing the potential difference necessary to overcome the true resistance of the electrolyte plus the potential difference required for overcoming the polarization potential difference at the electrodes. This difficulty is obviated by using the rapidly *alternating current from the secondary of an induction coil* instead of a steady current from a battery.

The time that the current continues in one direction is so short that no appreciable accumulation can form at the electrodes to produce an opposing difference of potential. An ordinary galvanometer would not be affected by an alternating current, but a *telephone* which is a very delicate detector of an alternating current may be substituted.

In the simplest form of apparatus a vertical glass tube of known cross section, which may be found by calipers (p. 14), holds the electrolyte. The electrodes are connected to wires that pass through the stoppers; the upper electrode can be raised or lowered as desired. The resistances corresponding to two different distances of separation of the electrodes should be determined. From the difference we get the resistance of a column whose length is the difference in the two lengths and thus eliminate uncertainty as to remaining polarization at the electrodes and the exact ends of each column.

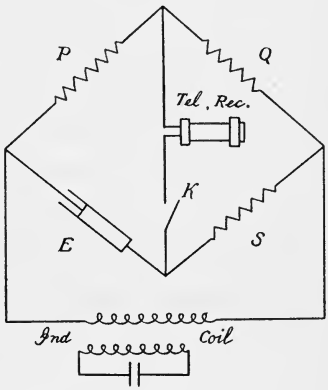

FIG. 59.

Two tubes should be available for the work. In one measurement should be made of the resistance of samples of several solutions of different concentrations, the samples being obtained from stock bottles. The other tube is for the purpose of determining the temperature coefficient of a solution. It is placed in a steam heater such as is used in Exp. XIX (p. 88). As the heating to a steady temperature will require considerable time, the tube should be prepared at the beginning of the work. The resistance of the electrolyte in it having been determined at room temperature, the heating may be allowed to proceed while the measurements with the other tube are made.

A tube of small cross section may also be used. If its

ends pass through stoppers into much larger tubes that contain the electrolyte and electrodes, the resistance measured will be virtually that of the electrolyte in the small tube. The diameter of the tube may be found as in Exp. XI (p. 58). Substituting a different length of the same tubing and taking differences we may as before eliminate residual polarization effects.

In measuring resistances it will probably be desirable to use equal resistances, e. g., 100 ohms, in the ratio arms of the bridge. It may be impossible to obtain a balance for which there is no sound, for even though there were a balance for steady current, there would not in general be a balance for varying currents such as are used in this experiment, owing to the inductive electromotive forces of capacity and self-induction in the resistance coils. When there is uncertainty as to whether a small resistance should be added or cut out, the ear is often assisted by adding and cutting out a larger resistance about which there is no doubt. On comparing the change of tone on a variation of this latter resistance with the variation of tone with the uncertain resistance, one can often decide whether the small resistance should be added or not. With a little practice one should determine resistances within 1 per cent.

Calculate the specific resistance of each solution at each temperature and tabulate the results. Find also the temperature coefficient of the solution which was heated and calculate its specific resistance at 0°.

Questions.

1. Why should we expect the resistance to decrease with increased temperature?
2. What is supposed to be the nature of electric conduction in an electrolyte?
3. Are the specific resistances inversely proportional to the concentrations? Why?

LVI. COMPARISON OF E. M. F.'S BY HIGH-RESISTANCE METHOD.

Watson's Practical Physics, pp. 430–431; *Stewart and Gee's Practical Physics*, II, pp. 101–102.

The readiest method of comparing the electromotive forces of cells is by means of a galvanometer of sufficiently high resistance. If the deflections are (by the use of added resistances) kept small the deflections of the galvanometer will be closely proportional to the currents that pass through it or $i=k.d$ where k is a constant. Two methods may be employed for comparing two cells. In the first, called the "equal resistance" method, the total resistance R is kept constant (the resistance of the cells being supposed negligible). Hence, by Ohm's Law, the e. m. f.'s are proportional to the currents, that is, to the deflections, or

$$\frac{E_1}{E_2}=\frac{d_1}{d_2}$$

In the other or "equal deflection" method, such resistances are used in the circuit that the cells cause equal deflections of the galvanometer. Hence by Ohm's Law, since the currents are equal, the electromotive forces must be proportional to the resistances, or

$$\frac{E_1}{E_2}=\frac{R_1}{R_2}$$

Both methods should be employed to find the e. m. f.'s of several cells by comparing them with that of a standard Daniell cell (p. 159). Directions for the adjustment of the telescope and scale are given on p. 25.

(A) *Equal Resistance Method.*—Make R such that the standard Daniell cell gives a deflection of about 10 cm. on a scale about 1 m. from the mirror. Make a reading of the zero; i. e., when no current passes through the galvanometer. Send the current through the galvanometer and read the division now on the cross-hair. In this way make at least six readings on one side and, reversing, make six on the

other. Read the zero often, as it is liable to change. In reading, use, if necessary, the method of vibration (see p. 23). If the vibrations are irregular on account of trolley currents or other disturbances, estimate the position of equilibrium from the vibrations without actually making readings. With some galvanometers the damping is so great that the system comes to rest instead of vibrating about the position of equilibrium. In this case the true reading can be made at once. Always, if possible, estimate tenths of the smallest divisions. When you have thus found the mean deflection for the standard, find similarly the deflection for as many different types of cells as time allows. With the other cells, three readings on a side will be sufficient. The internal resistance of the different batteries varies, but the differences are negligible compared with the total resistance of the circuit. Express in volts your final values of the e. m. f.'s of the cells tested.

(B) *Equal Deflection Method.*—With a resistance which gives a deflection of about 10 cm., make at least six careful readings of the deflection on each side given by the standard Daniell cell. Replace the standard by one of the cells to be tested and vary the resistance of the circuit until the deflection is the same as you found it on this side for the standard. Similarly find the resistance which will make the deflection on the other side the same as that given by the standard on that side. We can neglect, in comparison with the resistances in the boxes, the resistance of the battery and connecting wires, but not the resistance of the galvanometer. Take the mean of the two resistances determined above, plus the resistance of the galvanometer, as the resistance required to give the same deflection as the standard cell gave through the box-resistance used with it, plus the galvanometer resistance. The *resistance of the galvanometer, G,* must be determined as in Exp. XLV (last paragraph).

In determining the possible error of your results, estimate the possible error of resistances from the least change in

resistance which will have an appreciable effect, and the possible error of deflections from the mean deviation from the mean in your readings.

Questions.

1. What are the advantages and disadvantages of the type of galvanometer used in this experiment compared with other types used in the laboratory?
2. Which of the two methods do you consider the better? Why?
3. How could this method be used for finding the internal resistance of a cell?
4. Are the deflections of a galvanometer strictly proportional to the currents? Why?

LVII. COMPARISON OF E. M. F.'S AND MEASUREMENT OF BATTERY RESISTANCE BY CONDENSER METHOD.

Text-book of Physics (Duff), pp. 530–534, 562; *Watson's Physics*, pp. 634; *Watson's Practical Physics*, p. 526; *Henderson's Electricity and Magnetism*, pp. 185–187.

When a condenser of capacity C is connected to a battery of e. m. f. E it receives a charge $Q=CE$. If it be then connected to a ballistic galvanometer, the throw, d, will be proportional to Q, or $Q=K.d$, where K is a constant. We shall apply this to (A) compare e. m. f.'s and (B) measure the resistance of cells. We shall describe these separately, but in practice they may be combined.

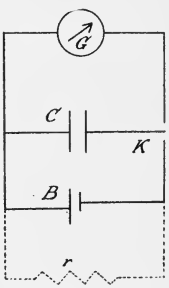

FIG. 60.

(A) Suppose the condenser is first charged by a battery of e. m. f., E_1, and the deflection when connected to the ballistic galvanometer is d_1, and suppose that when this same condenser has been charged by a battery of e. m. f., E_2, the deflection is d_2; then

$$Q_1=CE_1=Kd_1;\ Q_2=CE_2=Kd_2$$

$$\therefore \frac{E_1}{E_2}=\frac{d_1}{d_2} \text{ or } E_1=E_2\frac{d_1}{d_2}$$

Use a key with an upper and a lower contact. The condenser should be connected to the battery when the key is down and to the galvanometer when the key is up. Be very careful never to connect the battery directly to the galvanometer. When a discharge is sent through a ballistic galvanometer, the needle swings over to one side and then swings back. Observe the reading of the scale on the vertical cross-hair of the telescope when the needle stops and turns back. Always in such work estimate tenths of the smallest division. Before each throw bring the needle as nearly as possible to rest. The zero is likely to change;

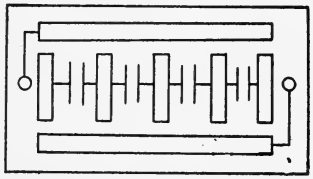

FIG. 61.

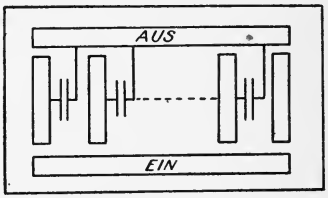

FIG. 62.

therefore, before each throw, record the zero, and, after each throw, record both the turning-point and the difference between this turning-point and the zero; i. e., the amount of the throw.

Always charge the condenser for approximately the same length of time, for instance, five seconds. With a standard Daniell cell (p. 159), record six throws on one side. Reverse the battery connections and record six throws on the other side. Let the mean of these be d_1. Replace the Daniell cell by one of another type and find as before the mean throw d_2. If E_2 is the e. m. f. of the latter cell

$$E_2 = 1.105\frac{d_2}{d_1}$$

Measure in this way the e. m. f. of as many cells of different type as possible.

(B) This method of finding the resistance of a cell depends on the fact that when the poles of a cell of resistance B are

joined by a conductor of resistance r, the difference of potential of the poles depends on the ratio of r to B. Let the current be i. Then the difference of potential of the poles is ri by Ohm's Law applied to the part r of the circuit. If in this condition the poles be joined to a condenser of capacity C it will receive a charge Cri and if this when discharged through a condenser causes a throw d'

$$Cri = K \cdot d'$$

Now if E be the e. m. f. of the cell, $E = (B + r)i$ by Ohm's Law applied to the whole circuit. Hence, if the condenser be charged by the cell when it is not short-circuited as above, the charge C E also equals $Ci(B + r)$ and if the deflection when the condenser is discharged is d,

$$Ci(B + r) = Kd$$

Dividing one equation by the other and solving for B,

$$B = r\frac{d - d'}{d'}.$$

The connections are the same as when comparing e. m. f.'s with the addition of a circuit containing a resistance and a very low resistance key (e. g., a mercury key), connecting the poles of the cell. The battery should be short-circuited just before the charging key is depressed, and the short-circuiting key should be released immediately after the other, otherwise the battery will run down. Choose such a short-circuiting resistance that the galvanometer throw is reduced to about half the value which it has without the short-circuit. Do not use the plug-box resistances for this work, on account of the danger of burning them out, but use open wound resistances of large wire.. Find the internal resistance of cells of several different types.

In estimating the possible error of your results, estimate the possible error of your mean readings from the mean deviation from the mean in the individual readings.

Additional exercises (to be performed if time permit.)

(C) Study the effect of *length of time of charge* by means

of the throws obtained with the condenser charged for different lengths of time with the same battery.

(D) Study the *leakage of the condenser* by comparing the throws when the condenser has been successively charged with the same e. m. f., and has remained charged for different intervals of time.

(E) Study the *electric absorption* of the condenser by charging for several minutes, discharging and reading the throw and immediately insulating; after one minute, again discharge and insulate. Continue discharging for several minutes, the condenser being insulated during the minute intervals.

Questions.

1. What are the peculiarities and requirements of a good ballistic galvanometer?
2. What is the construction of a condenser and what do absorption and leakage mean?
3. How could you find the resistance of the galvanometer used, employing a condenser and a known resistance?

LVIII. CALIBRATION OF A VOLTMETER.

Ames' General Physics, pp. 674–675; *Text-book of Physics (Duff)*, pp. 570–571; *Watson's Practical Physics*, pp. 493–498; *Henderson's Electricity and Magnetism*, pp. 200–208; *Hadley's Electricity and Magnetism*, p. 320.

A *voltmeter may be calibrated* by balancing a part of the e. m. f. applied to the terminals of the voltmeter against the e. m. f. of one or more standard cells. To do this a very high resistance circuit, consisting of a high resistance-box in series with an ordinary resistance-box, is placed in parallel with the voltmeter. The fall of potential in part of the low resistance-box is measured by a side-circuit consisting of the standard cell, a sensitive galvanometer and a key, the standard cell being so turned that it tends to send a current through the galvanometer in the opposite direction to the fall of potential in the box.

Let e be the e. m. f. of the cell, E the potential difference at the terminals of the voltmeter, r_1 the resistance across

which the galvanometer circuit is connected, and r_2 the remaining resistance in the high-resistance circuit. When r_1 is adjusted so that there is no deflection when the key is pressed

$$E = \frac{r_1 + r_2}{r_1} e$$

A special fuse-wire for very low currents should be placed immediately adjacent to the battery to prevent the possibility of injury to the resistance-boxes. The main circuit should be closed through a spring-key only a sufficient length of time to enable the voltmeter or the galvanometer to be read. A high resistance should be placed in series with the standard cell to prevent any considerable current passing through it. In first performing this experiment it is well to use a simple and inexpensive form of standard cell, and the Daniell cell (p. 159) will be suitable. For later and more accurate work, either the Clark or the Weston cell should be used. By varying the number of cells in the main circuit or using different resistances in the main circuit, different voltages at the terminals of the voltmeter may be obtained.

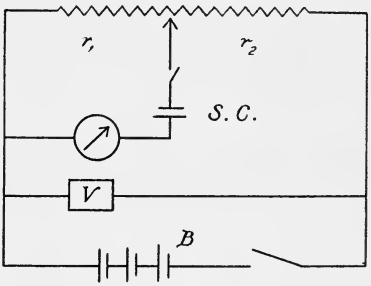

FIG. 63.

The above method will not apply if the voltmeter is to be tested at voltages less than the e. m. f. of the standard cell. In this case, an inversion of the connections may be used. Instead of balancing a variable part of the voltage against the e. m. f. of the cell a variable part of the e. m. f. of the cell is balanced against the voltage. A little consideration will indicate the necessary change of connection.

Instead of the above temporary arrangement of circuits, a *Potentiometer*, which consists essentially of the several

circuits with the necessary resistances and keys in permanent connection, may be used. With its aid the work may be performed more rapidly and more accurately. Its parts and connections should be carefully traced out with the assistance of a large diagram (which may be attached to the wall near the instrument) and additional explanation will be supplied by the instructor.

A calibration curve, consisting of true volts plotted against scale readings, should be drawn.

Questions.

1. Prove the above formula by applying Kirchoff's laws.
2. Draw a diagram showing the connections when a millivoltmeter has to be calibrated.
3. Draw a diagram to show how the above method could be adapted to compare the *e. m. f.'s* of cells.

LIX. CALIBRATION OF AMMETER.

Hadley's Electricity and Magnetism, p. 325; *Watson's Practical Physics*, pp. 516–517; *Henderson's Electricity and Magnetism*, p. 205.

A method somewhat similar to that used for the voltmeter may be employed. The current from a storage battery that passes through the ammeter passes also through a conductor of large current capacity and of measured resistance and a switch. The potential difference at the ends of this conductor is found by a shunt circuit, consisting of a high resistance-box in series with a box containing low resistances. In parallel with the latter is a circuit containing a Daniell cell, a sensitive galvanometer and a key. A special very fine fuse-wire should be used in series with the two boxes, and its resistance should be known and taken account of in calculating the current. The large conductor should be immersed in oil and its temperature kept as nearly as possible at the temperature at which its resistance is determined. To prevent heating, the main current should be closed for short intervals only.

The galvanometer should be protected by a shunt during the first adjustments. Notice first in which direction the galvanometer moves when the key in its circuit is depressed. The deflection should be reversed or reduced when in addition the switch is closed. If this is found not to be so, the connection of either the Daniell cell or the storage batteries should be reversed. The resistance in the box nearest the galvanometer and, if necessary, in the other box also, should be varied until there is no deflection if the galvanometer key is depressed when the switch is closed.

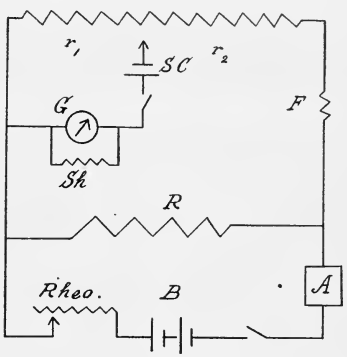

Fig. 64.

When the adjustment has been obtained as closely as possible, the fall of potential between the points of the high-resistance circuit to which the standard cell circuit is attached equals the e. m. f. of the cell (p. 159). From this and the resistances of the boxes the fall of potential between the ends of the large conductor is found and then from the resistance of the large conductor the current through it and the ammeter is calculated. The total resistance in the two boxes must be kept high. A preliminary calculation will show about how large the resistance of large capacity should be. A number of currents distributed over the range of the ammeter should be used and from the results a calibration curve should be drawn.

In the above we have assumed that the voltage applied to the conductor exceeds that of the cell. If the reverse is the case, the arrangement must be inverted; i. e., part of the e. m. f. of the cell must be balanced against the voltage applied to the conductor. This method must be applied when the current is less than the quotient of the e. m. f. of the cell and the resistance of the conductor.

Instead of the arrangements of circuits above, the Potentiometer referred to in Exp. LVIII may be used to measure the fall of potential in the conductor of large current capacity.

Questions.

1. Why must the resistance in the shunt circuit be large?
2. Storage batteries giving an e. m. f. of 50 volts are available for calibrating an ammeter whose range is from 5 to 25 amperes and whose resistance is 0.2 ohms. (a) What is the least possible value for the resistance of large capacity? (b) What is the greatest value?

LX. COMPARISON OF CAPACITIES OF CONDENSERS.

Hadley's Electricity and Magnetism, pp. 331–334; *Henderson's Electricity and Magnetism*, pp. 235–241; *Watson's Practical Physics*, pp. 530–535.

Two or more condensers are to be compared by three methods.

(A) *First Method.*—Each condenser is charged in turn by the same battery and then discharged through a ballistic galvanometer. Let the capacities of the two condensers be C_1 and C_2. The charges which they receive when connected to a battery of e. m. f. E, are $Q_1 = C_1 E$, and $Q_2 = C_2 E$. Let the throws of the galvanometer when the condensers are discharged through it be d_1 and d_2, respectively. Then

$$\frac{q_1}{q_2} = \frac{C_1}{C_2} = \frac{d_1}{d_2}$$

(Exp. LVII). The connections are the same as in Exp. LVII, with the addition of one or more keys to charge alternately two or more condensers. If either deflection be too small, additional cells should be added. Storage cells may be used if connections are made through special very fine fuse-wires to protect the resistances.

If either deflection be too large the *galvanometer should be shunted* by a known resistance, S. Let G be the resistance of the galvanometer determined as in Exp. XLV (last paragraph), d' the throw obtained with the galvanometer shunted, d the throw which

would have been obtained without the shunt, q' the quantity of electricity passing through the galvanometer, q'' the quantity passing through the shunt. Then

$$\frac{q''}{q'}=\frac{G}{S}$$

for charges of electricity, like steady direct currents, divide inversely as the resistances (p. 158). Hence

$$q=q'+q''=q'(1+\frac{G}{S}).$$

Since

$$\frac{q}{q'}=\frac{d}{d'},$$

$$d=d'(1+\frac{G}{S}).$$

(B) *Bridge Method.*—The two condensers to be compared, C_1 and C_2, form two arms of a Wheatstone's Bridge, two high non-inductive resistances, R_1 and R_2 (see figure), preferably several thousand ohms, forming the other two arms. These two resistances are adjusted until on closing the battery circuit at a the galvanometer is not disturbed. Then during both charge and discharge the farther poles of the condenser (A and B) must remain at the same potential as well as the nearer poles (joined at D). Hence the charges Q_1 and Q_2 in the condensers must have the ratio, $Q_1 : Q_2 :: C_1 : C_2$. But the quantities which have flowed into the condensers will be inversely proportional to the resistances through which the charges have flowed, that is, $Q_1 : Q_2 :: R_2 : R_1$. Hence

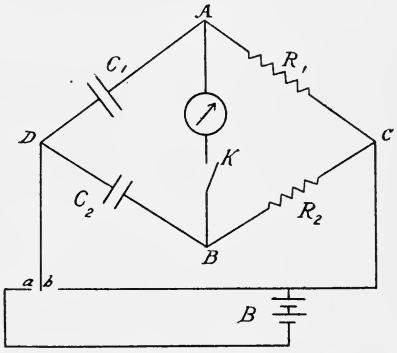

FIG. 65.

$$\frac{C_1}{C_2}=\frac{R_2}{R_1}.$$

The battery key has an upper and lower contact; the upper contact (b), against which the lever ordinarily rests,

short-circuits the battery terminals of the bridge, thus keeping the condenser uncharged. The sensitiveness may be increased by increasing the number of cells in the battery, and also by using a double commutator (see p. 161). Instead of a galvanometer a telephone may be used in this method, the battery being replaced by a small induction coil.

(C) *Thomson's Method of Mixtures.*—The connections are as shown in the figure. K_1 is a Pohl's commutator, K_2 an ordinary single contact switch. When the swinging

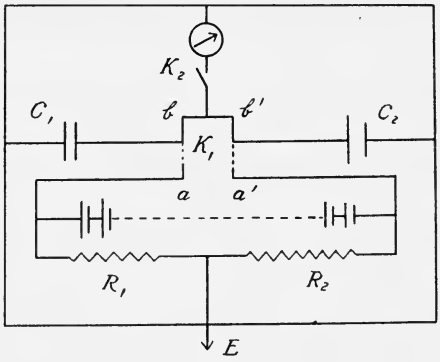

FIG. 66.

arm of the commutator is in the position aa', the two condensers are charged, C_1 to the difference of potential at the extremities of R_1, C_2 to the difference of potential at the extremities of R_2. The swinging arm of the commutator is now placed in the position $b'b'$ and the two charges are allowed to mix. If they are exactly equal, being of opposite sign, the galvanometer will not be affected when K_2 is depressed. R_1 and R_2 (which are large resistances, preferably several thousand ohms), are adjusted until this is secured. The charges being equal, $C_1V_1 = C_2V_2$, and since $V_1 : V_2 :: R_1 : R_2$,

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}.$$

Questions.

1. What is the composite capacity of three microfarad condensers in parallel? In series?
2. Which of these three methods do you consider best?
3. State briefly in words (without formulæ) why charges divide like steady currents; i. e., inversely as the ohmic resistances.
4. Why cannot *series* resistance be used in (A) to reduce the sensitiveness of the galvanometer.

LXI. ABSOLUTE MEASUREMENT OF CAPACITY.

The magnitude of a capacity can also be found without the use of a known capacity with which to compare it. This can be done in different ways. The following is one of the simplest.

Let Q be the charge received by the condenser of unknown capacity C when connected to a cell of known e. m. f., E. Then

$$C = \frac{Q}{E}$$

To find Q discharge the condenser through a ballistic galvanometer the constant of which has been found by the method of Exp. LXIV. If the constant be K and the deflection D, $Q = KD$.

LXII. COEFFICIENTS OF SELF-INDUCTION AND OF MUTUAL INDUCTION.

Text-book of Physics (*Duff*), pp. 609–611; *Ames' General Physics*, pp. 743–745; *Watson's Practical Physics*, pp. 543–548; *Parr's Practical Electrical Testing*, p. 207; *Hadley's Electricity and Magnetism*, pp. 417–422.

The *coefficient of self-induction* of a circuit is the number of magnetic lines of force which link with the current when the circuit is traversed by unit current. Owing to the difficulty of calculating this important quantity from the dimensions of the circuit, experimental methods of determination have much value.

(A) Probably the best method (using direct currents) is *Anderson's Modification of Maxwell's Method.* The connections are shown in the figure. The coil of self-induction L and resistance Q is made one arm of a Wheatstone's Bridge (preferably Post-office Box form). Obtain a balance for steady currents by proper variation of S, so that when K_1 is closed and then K_2, the galvanometer is not disturbed. For delicacy of adjustment it is well to either have a resistance which can be varied continuously form a part of S, or make the ratio arms P and R such that S is large. Vary r, the resistance in the battery circuit, and if necessary, vary the capacity of the condenser C until there is a balance for transient currents; i. e., until the galvanometer is not disturbed when K_2 is depressed and K_1 depressed afterward. Then if C is the capacity of the condenser.

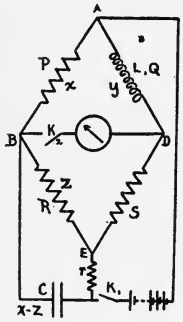

FIG. 67.

$$L = C[r(Q+P)+PS].$$

For at time t let x = current in branch AB, y = current in AD = current in $D\,E$, z = current in BE $\therefore$ current in $r = y+z$. q = charge in condenser, e = potential difference of its poles $= Rz + r\,(y+z)$.

Since there is a balance for transient currents we may equate the e. m. f. in AD to that in AB. Hence

$$Qy + L\frac{dy}{dt} = Px.$$

The current in the branch containing the condenser is $(x-z)$; but it can also be expressed as

$$\frac{dq}{dt} \text{ or } C\frac{de}{dt}.$$

Hence

$$x - z = C\left[r\frac{dy}{dt} + (R+r)\frac{dz}{dt}\right].$$

Now since there is a balance for steady currents $RQ = PS$ and since $Rz = Sy$, it readily follows that

$$Qy + L\frac{dy}{dt} = Qy + C[r(Q+P)+PS]\frac{dy}{dt}$$
$$\therefore L = C[r(Q+P)+PS].$$

If the resistances are expressed in ohms and the capacity in farads, the results will be in henries.

Measure the self-induction of a coil whose length is great compared with its diameter and compare the result with that calculated. To *calculate* the coefficient of self-induction it is necessary to know the number of lines of force passing through the coil. This number multiplied by the number of turns will give the number which link with the current; i. e., the self-induction. If A be the area of the cross section of a solenoid of practically infinite length, with n_0 turns per cm. of length, the number of lines is $4\pi n_0 A$ for unit current. The number of turns in a length d is $n_0 d$; hence the coefficient of self-induction of this length is $4\pi A n_0^2 d$ in C. G. S. units. Reduce to henries by dividing by 10^9 (p. 162).

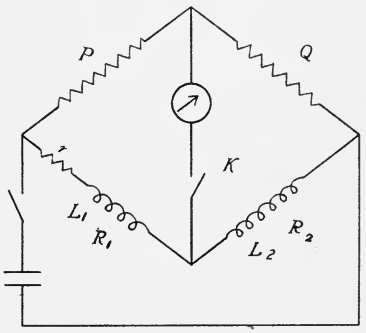

FIG. 68.

To secure greater sensitiveness in making the balance for transient currents, replace the battery by the secondary of a small induction coil and the galvanometer by a telephone, or use a double commutator (see p. 161).

(B) *Comparison of Two Coefficients of Self-induction.*— The two coils of self-induction L_1, L_2, and resistances R_1, R_2, are placed in two arms of a Wheatstone's Bridge, a variable resistance, r, being included in one arm. By varying r, and, if possible, by varying one of the self-inductances, if not, by varying r, P, and Q, find a balance for both steady and transient currents.

Then for steady currents

$$\frac{R_1 + r}{R_2} = \frac{P}{Q},$$

and for transient currents

$$\frac{\sqrt{(R_1+r)^2+\omega^2 L_1^2}}{\sqrt{R_2^2+\omega^2 L_2^2}}=\frac{P}{Q},$$

where $\omega = 2\pi \times$ frequency. Hence

$$\frac{L_1}{L_2}=\frac{P}{Q}.$$

The above adjustment is obtained by first securing a balance for steady currents. A balance for transient currents is then sought by varying L_1 or L_2. If this cannot be secured, r and P must be both increased or decreased and a balance for steady currents again obtained and then one for transient currents found by varying L_1 or L_2. If neither L_1 nor L_2 can be varied, a balance can only be obtained by a series of trials as above, the ratio of P and (R_1+r) being kept constant, so that the steady current balance may not be disturbed. To increase the sensitiveness with transient currents, a double commutator (p. 161) should be used or the battery may be replaced by the secondary of an induction coil and the galvanometer by a telephone.

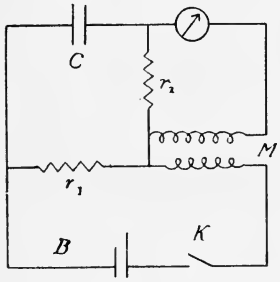

FIG. 69.

(C) The *coefficient of mutual induction* of two coils is the number of lines of force which link with the turns of the other when the first is traversed by unit current. *Pirani's method* is perhaps the most satisfactory for the experimental determination of coefficients of mutual induction. The connections are shown in figure 69. If M be the required coefficient, C the capacity of the condenser, and r_1 and r_2 the values of the variable resistances for which the galvanometer is not disturbed,

$$M = Cr_1r_2.$$

For let the steady current in the battery circuit $= i$. The potential difference at the terminals of r_1, $= ir_1$. The charge of the condenser is Cir_1. If t = time required to establish or destroy the battery current the average current in the condenser branch during this time = Cir_1/t and the potential difference at the terminals of $r_2 = Cir_1r_2/t$. Opposing this e. m. f. in the galvanometer circuit is that due to M, the average value of which $= Mi/t$. If the galvanometer is not disturbed on making or breaking the battery circuit, $Cir_1r_2/t = Mi/t$ $\therefore M = Cr_1r_2$.

The secondary of M is acted on by two e. m. f.'s—one due to its connection with the main circuit in which there is an e. m. f., the other due to mutual induction between the primary and secondary. If these two do not oppose one another a balance cannot be found. If such is found to be the case the connections of the secondary to the galvanometer circuit must be reversed. To increase the sensitiveness of the method a double commutator may be used or, better still, the battery and galvanometer may be replaced by a small induction coil and telephone.

When an approximate adjustment has been found C and r_1 should be altered until the sensitiveness is a maximum, and then r_1 and r_2 treated in the same way.

Find the coefficient of mutual induction of two coils, one wound upon the other, and one of which is long compared with its diameter, and compare the result with that calculated from the definition.

QUESTIONS.

1. Coils with iron cores do not have definite induction coefficients. Explain.
2. How are resistance coils in boxes wound so as to be free from self-induction?

LXIII. STRENGTH OF A MAGNETIC FIELD BY A BISMUTH SPIRAL.

Ames' General Physics, p. 758; *Hadley's Electricity and Magnetism*, p. 296.

The electrical resistance of a bismuth wire is changed when it is placed transverse to a magnetic field and the magnitude of the change depends on the strength of the field.

When a curve representing the resistance of a flat spiral of bismuth as a function of the strength of the magnetic field has been obtained the spiral may, in connection with a Wheatstone's Bridge, be used to measure the strength of any magnetic field within the range of the calibration. For instance, it may be used to study the magnetic field of an electromagnet. The following three points may be examined:

(A) Find how the magnetic field between the poles varies when the strength of the current actuating the electromagnetic is varied by means of a rheostat.

(B) Find how the strength of the field midway between the pole-pieces changes when the distance apart of the pole-pieces is varied, the current being kept constant.

(C) Find how the strength of the field in an equatorial plane varies with the distance from the axis of the pole-pieces.

In each case represent the results by means of a curve.

LXIV. CONSTANT AND RESISTANCE OF A BALLISTIC GALVANOMETER.

Text-book of Physics (*Duff*), pp. 556, 562; *Ames' General Physics*, p. 674, 712; *Watson's Practical Physics*, pp. 518–524; *Pierce, Am. Acad. Arts and Sc.* Vol. 42, pp. 159–160.

When quantities of electricity are discharged through a ballistic galvanometer (p. 156) the throws are proportional to the quantities or,

$$Q = K.d,$$

where K is the constant of the ballistic galvanometer. To determine the value of K a known quantity must be discharged through the galvanometer and the throw noted.

This known quantity might be obtained from a condenser of known capacity, charged to a known potential, or by turning an earth inductor (Exp. XLIII) in a field of known strength. Both of these methods, however, require that other constants (capacity and e. m. f. in the first, strength of field in the second) be determined.

A simpler method is to use a so-called calibrating-coil; i. e., an induction coil of known winding without a magnetic core. The primary is a long, straight helix, so long that there is no appreciable leakage near the center. Over the center there is wound a secondary. If the primary be of n turns per cm. and the secondary be of n' *total* turns, then the magnetizing force produced by a current of i amperes in the primary is

$$h = \frac{4\pi ni}{10},$$

and the quantity induced in the secondary by making or breaking i is

$$q = \frac{han'}{10^8 r} = \frac{4\pi nn'ai}{10^9 r},$$

where a is the area of cross section of the cylinder on which the primary is wound, r is the total resistance of the secondary circuit and the factor 10^8 is required when q is in coulombs and r in ohms (p. 162).

The number of lines of force that pass through the secondary is $a\,h$. Hence when h is increasing the induced e. m. f. is in absolute units

$$e = -n'\frac{d(ah)}{dt}.$$

The quantity induced is $\int i\,dt$ and i equals e/r. Hence

$$q = \int \frac{n'}{r}\frac{d(ah)}{dt} = \frac{n'ah}{r}.$$

From the above expression for q and the throw d, K can be calculated. If the throw is small it may be doubled by reversing i and the half of the double throw taken for d. Several currents should be tried. The value of K thus found is in coulombs per scale division. If the distance of the scale from the galvanometer be changed in any proportion K will be increased in the same proportion. Hence the distance should be noted unless it is not liable to change.

It is often necessary to change the sensitiveness of a ballistic galvanometer by shunting it or putting resistance in series

with it. To allow for this we must know the resistance of the galvanometer.

The resistance of a ballistic galvanometer when used ballistically on a closed circuit is different from its resistance when used as an ordinary galvanometer. This is due to the fact that the galvanometer coil moves in a magnetic field, and thus an induced e. m. f. is produced, which opposes the applied e. m. f. and has the same effect as an added resistance. To find the effective resistance of a ballistic galvanometer we may use the same apparatus and connections as in finding the constant of the galvanometer. If a current be reversed in the primary of the calibrating coil, the quantity of electricity that will flow through the secondary will vary inversely as the total secondary resistance. Hence, by observing the throw with a certain primary current, and then increasing the secondary resistance by the insertion of a box-resistance and repeating the reversal of the primary, we can, by proportion, find the resistance of the galvanometer when used ballistically.

The resistance of the galvanometer should also be found by the method of Exp. XLVI or that of Exp. XLV and compared with the above.

LXV. MAGNETIC PERMEABILITY.

Text-book of Physics (*Duff*), pp. 500–503; *Ames' General Physics*, pp. 609, 615; *Watson's Physics*, pp. 712–714; *Hadley's Electricity and Magnetism*, pp. 384–392; *Watson's Practical Physics*, pp. 563–568; *Henderson's Electricity and Magnetism*, pp. 282–284; *Ewing's Magnetism in Iron*, Chapter III.

A current in a long solenoid of wire will produce near the center of the solenoid a magnetic force H, which may be specified by the number of lines of force per unit of area at right angles to the lines. If a long iron rod be now thrust into the solenoid, the number of lines of force (now called lines of induction) will be much greater, say B per unit of area. The permeability of the iron is defined as $\mu = B \div H$.

If this experiment were performed with comparatively short iron rods, it would be found that B would be less the shorter the rod. One consistent way of explaining this is to consider the free poles developed at the ends of the rod when magnetized. A little consideration will show that they of themselves would produce a magnetic force in the space occupied by the iron, this magnetic force being opposed to the original magnetizing force, and so we may say that the effective magnetic force, H, is the original magnetic force diminished by the demagnetizing force of the poles. It is this effective magnetic force that we should divide into the induction to get the permeability. The calculation of the demagnetizing force is usually difficult and uncertain, and so it is better to take some method of eliminating it.

One such way is that implied in the statement at the outset, to use a long rod, for that will diminish the magnitude of the demagnetizing force at the center. But the necessary length makes it inconvenient to test specimens in this way. Another method is to join the ends of the rod by a heavy yoke of iron, for opposite poles developed in the yoke neutralize the effect of the poles in the rod. (This is one way of stating the case. Another way is to say that the yoke carries around the lines of force. A third way is to say that the yoke diminishes the magnetic resistance of the circuit.) The difficulty with a yolk method is in getting a satisfactory contact between yoke and rod. A very small gap will result in the neutralization being not quite complete (or in leakage of lines of force or in magnetic resistance).

A more satisfactory method is to take an endless specimen; i. e., a ring. Then there are no free poles and no demagnetizing force. On the ring a magnetizing coil of N turns per cm. is wound. When a current of I amperes passes through it, the magnetizing force produced is

$$H = \frac{4\pi N}{10} I \quad . \quad . \quad . \quad . \quad (1),$$

For finding the value of B a secondary coil is wound on the

ring and put in series with a ballistic galvanometer. Suppose the iron initially free from magnetism. The setting up of the field B produces a discharge, Q, of electricity through the secondary. If A be the area of cross section of the ring and N' the total number of turns,

$$Q = \frac{N'AB}{R10^8}$$

R being the total (ohmic) resistance of the secondary circuit. (The factor 10^8 is not necessary if Q and R are in absolute units. It must be used when Q is in coulombs and R in ohms (p. 162). If the throw of the galvanometer is D

$$Q = K \cdot D$$

when K is the ballistic constant (Exp. LXIV). If K is not known, a calibrating coil for determining it should be included in the arrangement of the apparatus. From the above formulæ and the data, H, B and μ can be calculated.

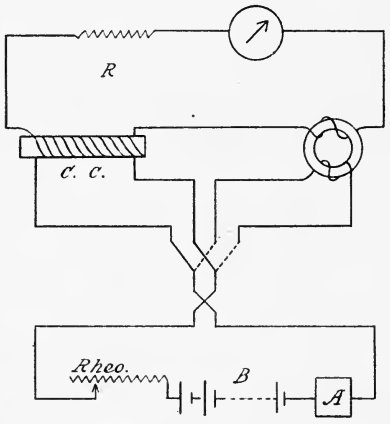

FIG. 70.

In making the connection for the practice of the method, it is much better to have a clear understanding of the plan and purpose of each part and to proceed systematically than to copy the connection from a diagram. In the first

place, the secondaries of both coil and ring should be kept permanently in series with the galvanometer. Then a switch is to be so arranged that the current can be passed through either the primary of the calibrating coil or that of the ring. A suitable rheostat and ammeter are needed in the primary circuit. If, as the primary current is increased, the deflections of the galvanometer become too great to be read, a resistance must be put in series or in parallel with the galvanometer. The former is preferable. In choosing this added series resistance, it is well to so choose it that the whole new secondary resistance is made a simple multiple of the former resistance. If this is done the throw will be reduced in the proportion in which the resistance is increased, and all throws may be reduced to what they would have been with the original resistance by multiplying the actual throw by the proportion in which the secondary resistance was increased. For methods of bringing the galvanometer to rest, see p. 156.

Before readings are begun the ring should be demagnetized as thoroughly as possible. This can be done by passing an alternating current through the primary and reducing it from a large value to zero by means of a rheostat, or, by rapidly commutating and at the same time reducing a direct current. *Also at each new value of the magnetizing current, before readings are taken, the commutator should be reversed several times,* so that the iron may come to a steady cyclical state. Instead of attempting to get the throw on making the primary current, the double throw on reversing the current is taken with both calibrating coil and ring and divided by 2.

At least three throws that agree well should be read for each strength of the primary current. The magnetizing current should be increased at first by small steps to bring out the characteristic features of the curve of magnetization, afterward by larger steps. The work need not be continued after the readings begin to differ in a much smaller proportion than the successive magnetizing currents, for this shows

approaching saturation. *The throw at break of current should also be carefully noted as a means of estimating the permanent magnetism;* for from the throw at break the diminution of B, and, therefore, the residual value of B, can be calculated as above.

In the report the various values of I, H, Q, B, and μ should be tabulated and a curve drawn with B as ordinates and H as abscissæ (B-H curve or curve of magnetization). On the same sheet a B-μ curve should also be drawn and a third curve showing the permanent magnetism as deduced from the throws at break of the current.

Questions.

1. Why is only the ohmic and not the self-inductive resistance of the secondary considered?
2. What is the effect of the windings being closer together on the inside of the ring than on the outside?
3. What is meant by intensity of magnetization? Susceptibility? Calculate a few values from your results.

LXVI. MAGNETIC HYSTERESIS.

Text-book of Physics (*Duff*), pp. 503–504; *Watson's Physics*, pp. 716–722; *Hadley's Magnetism and Electricity*, pp. 393–395. *Watson's Practical Physics*, p. 561; *Henderson's Electricity and Magnetism*, p. 294; *Ewing's Magnetism in Iron*, Chapter V.

Let a magnetizing force applied to a specimen of iron as in the preceding experiment be increased step by step and let the resulting increases of magnetization be observed. At some stage let the process be stopped and then the magnetizing force decreased by the same steps. It will be found that the steps of decrease of magnetization are less than those by which it at first increased, or the magnetization *lags* behind the magnetizing force. This is called *hysteresis*. For a complete view of the process a cycle must be completed, i. e., the magnetizing force must be decreased step by step to zero, then increased to a negative value equal (numerically) to the positive value at which the decreases were begun, then decreased again to zero, and finally

increased again to the highest positive value. Thus *a hysteresis loop will be obtained.*

With a ring specimen, over which primary and secondary coils are wound, there are two methods of procedure.

(A) *Step by Step Method.*—This method follows closely the general description given above. The successive steps are indicated in figure 71. The increases or decreases of I must be made without break of the current. The steps must not be too large or the points on the curve will be too far apart, and they must not be too small or the work

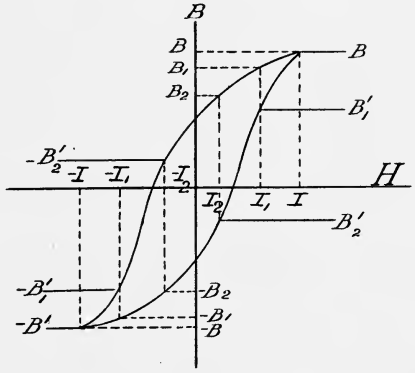

FIG. 71.

will become tedious. To satisfy these conditions, place in the primary circuit a special rheostat consisting of suitable resistances in parallel, each of which can be short-circuited by a knife-edge switch. Such a rheostat may be made up with resistances permanently connected in position, but a better plan is to use removable resistances. In the latter case a considerable collection of units should be supplied, and from these, by a preliminary trial, units that will produce suitable changes of I (e. g., from 4 to 0. 5 amp. by steps of 0. 5 amp.) should be chosen and placed in position in the rheostat.

It is not necessary to start from zero magnetization.

Beginning with the highest current to be used, reverse several times to produce a cyclical state and then find the throw on reversal. From this the maximum value of B can be calculated as in Exp. LXV. Then diminish the current by steps and note the throw in each step. After the step that reduces the current to zero, the current must be reversed and the resistances *increased* step by step. The rest of the process needs not be described. From each throw the corresponding change of induction, ΔB, is calculated as in Exp. LXV. When the cycle has been completed the algebraic sum of the throws should be zero. It should not be necessary to change the sensitiveness of the galvanometer; it will give the smaller throws with less accuracy, but they are less important. This "step by step" method of measuring hysteresis is the most instructive and is not difficult after some initial practice. It has, however, the disadvantage that an error in one reading of the galvanometer vitiates the whole.

(B) *The Ewing-Classen Method.*—The last-mentioned disadvantage is avoided in the method by starting each step from the maximum value of B. As before, we first find by reversals the value of B corresponding to the maximum value of I. We then diminish I (without breaking the current) and from the throw we calculate the diminution of B. This gives us a second point on the curve. We then return to the maximum current and, *after several reversals*, to re-establish the cyclical state, we again decrease I, but by a larger amount than before. From the throw we again calculate the diminution of B and thus get another point on the curve. Proceeding in this way, we reach the stage at which I is decreased from its maximum to zero. This gives us the point at which the curve crosses the axis on which B is plotted.

A simple method of producing the above changes of I is to connect the rheostat described under (A) in parallel with one of the cross-bars of the Pohl's commutator used for reversing I (Fig. 72). If this cross-bar be suddenly removed, the

resistance in the rheostat will be thrown into the circuit without breaking the current.

By the above process, we have obtained that part of the descending branch of the hysteresis loop, which lies to the right of the B axis. To obtain the remainder of the branch, we again proceed by steps from the positive maximum value of B, but, since each change of I will carry it from its positive maximum to a smaller negative value, we must simultaneously diminish and reverse the current. To be able to do this, remove the cross-bar of the commutator which is in parallel with the rheostat and turn the commutator so that the

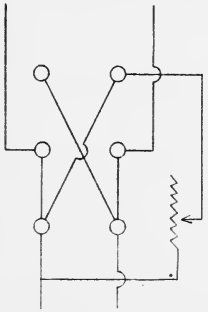

FIG. 72.

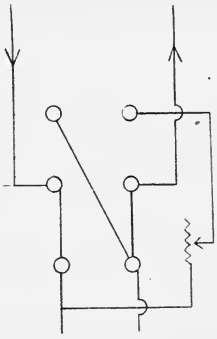

FIG. 73.

current flows to the ring, but does *not* pass through the rheostat (Fig 73). If the commutator be now reversed, the current will be reversed and will be diminished by passing through the rheostat. Thus we get another point on the curve and, by a series of such steps with decreasing resistances in the rheostat, the descending branch of the loop is completed. To trace the other branch we might proceed as above, beginning each step from the *negative* maximum of B. This, however, is unnecessary, since we would evidently be merely repeating the previous readings. The loop is symmetrical about the origin, and the co-ordinates of the ascending branch

are equal to those of the descending branch but with signs reversed.

It can be shown that the energy expended in such a cyclical change of magnetization is

$$\frac{1}{4\pi}\int HdB$$

ergs per c.c. of the iron. The integral also represents the area of the loop, due allowance being made for the scale on which it is plotted. Hence if the area be found by means of a planimeter (the use of which will be explained by an instructor), the energy loss per c.c. per cycle can be calculated.

The total number of lines of induction through each turn of the magnetizing coil is AB. Since the total number of turns is lN, when B is being increased there is induced in the magnetizing coil an e. m. f.

$$E=-\frac{d(lNAB)}{dt}=-VN\frac{dB}{dt}\text{ C. G. S. units.}$$

V being the volume of the core ($=lA$). The work done by the battery in time dt in overcoming this opposing e. m. f. is

$$dW=IEdt=INVdB\text{ ergs}$$

Now the area of the hysteresis loop is the integral of HdB and

$$HdB=4\pi NIdB$$

$$=\frac{4\pi}{V}dW$$

$$\therefore W=\frac{V}{4\pi}\int HdB$$

Questions.

1. What rise of temperature would 1000 cycles produce in the iron if no heat were lost?
2. How much less would the energy loss be if the maximum magnetization were half as great as in your cycle?

LXVII. (A) THE MECHANICAL EQUIVALENT OF HEAT. (B) THE HORIZONTAL INTENSITY OF THE EARTH'S MAGNETISM.

Text-book of Physics (*Duff*), pp. 557, 587–588; *Ames' General Physics*, pp. 664–665, 688; *Watson's Physics*, pp. 692–693, 674–677, 775–776; *Crew's General Physics*, §§ 289, 319; *Hadley's Electricity and Magnetism*, pp. 336–340, 458; *Watson's Practical Physics*, pp. 508–512.

If Q calories of heat be produced in a conductor by the passage of a current i for time t, and if no other work, chemical or mechanical, be performed, then

$$JQ = i^2Rt,$$

J being the mechanical equivalent. If i be expressed in amperes, R in ohms and Q in calories, i^2Rt will be in joules (one joule being 10^7 ergs), and J will be obtained as the number of joules in a calorie.

Q can be measured by immersing the conductor of resistance R in a known mass of water contained in a vessel of known water equivalent. The mass of water may be obtained with sufficient accuracy by measuring it from a burette. To eliminate the effects of radiation, conduction and convection, the water should be at the beginning of the passage of the current as much below the temperature of the room as it finally rises above it, for the current is kept steady and the temperature of the water therefore rises steadily, so that it is as long above the room temperature as below.

The resistance R may be measured against a standard ohm coil by Wheatstone's Bridge, and, since it will be found necessary to use a wire of comparatively small resistance, R should be measured with great care. Leads of large size and small length should be employed for connecting the wire to the bridge. While being measured it should be immersed in the calorimeter in water at the temperature of the room, so that the mean resistance throughout the experiment is obtained. To reduce to absolute units the resistance in ohms is multiplied by 10^9.

The current, i, may be obtained from its chemical effect in another part of the circuit. Careful measurements have shown that unit current (C. G. S.) flowing through a solution of copper sulphate of a certain strength between copper electrodes deposits 0.00326 gms. of copper per second on the cathode.

The form of *copper voltameter* employed consists of a glass vessel containing a solution of copper sulphate into which dip three plates. The two outer are of heavy copper

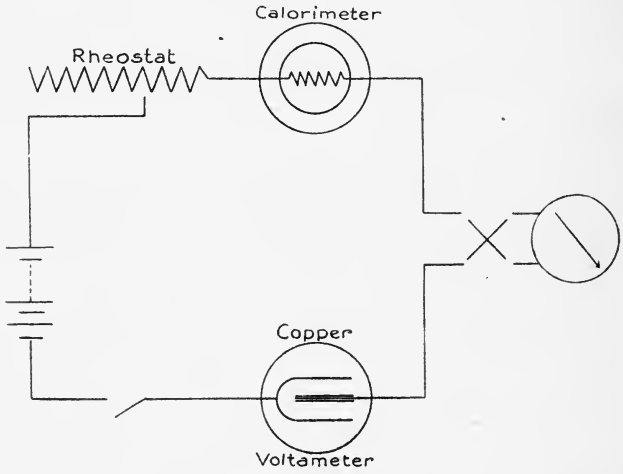

FIG. 74.

and are both joined, directly or indirectly, to the positive pole of the battery, forming the *anode.* The intermediate plate is thin and light, and is connected to the negative pole of the battery, forming the *cathode.* A satisfactory *solution* consists of 15 grams of copper sulphate dissolved in 100 grams of water, to which are added 5 grams of sulphuric acid and 5 grams of alcohol. (The alcohol is easily oxidized, thus preventing the oxidization of the deposit on the cathode and the formation of polarizing compounds at the anode.)

Clean the two anode plates with sand-paper and fasten them in the two outside binding posts of the top of the voltameter. Clean with sand-paper a cathode plate, wash with tap water and then with alcohol. When dry, weigh on one of the chemical balances, weighing to milligrams with the rider. Wrap in paper and set aside. Be very careful not to touch with the fingers any part of the plate which will be in the solution, after it has been cleaned. Clean with sand-paper a trial cathode and mount it on the middle binding post. Before putting the voltameter in the circuit, dip the two wires which are to be connected to the voltameter in the solution of the voltameter. Decrease the variable resistance until you have a moderate current, but do not entirely cut it out. Notice on which wire copper is deposited as a brown powder. Connect this wire to the cathode of the voltmeter and the other wire to the anode plates.

It is important that the current be kept constant. It is true that even if the current vary, the deposit will give the true mean value of the current. But what is needed is the mean value of i^2, and this is not necessarily the same as the square of the mean value of i. If a storage battery in good condition be used as the source of current, the current will not vary much; nevertheless, a tangent galvanometer or an ammeter should be included in the circuit to test the constancy of the current. There is also another reason for including a current meter of some form. The difference of potential at the terminals of the heating coil, or iR, must not be as great as the e. m. f. (1.6 V) that will electrolyze water, otherwise some part of the energy of the current will be spent in chemical work. Knowing R, one can choose a safe value for i. If the constant of the galvanometer be not known, it can be calculated roughly from the dimensions of the coils and the approximate value (say 0.18) for the horizontal component of the earth's field (see Exp. XLII), and so the deflection corresponding to a safe value of i deduced. An ammeter, if available, affords a still simpler means.

If a *tangent galvanometer* be used a fairly reliable value for the horizontal component of the earth's field may be deduced from the results of the experiment. For this purpose the dimensions of the galvanometer should be carefully measured and the current through it frequently reversed and carefully read. The *Helmholtz form of tangent galvanometer* may be used. This consists of two coils separated a distance equal to their common radius, with the needle on their common axis midway between them. This arrangement of two coils produces a very uniform field over quite an area where the needle is located, allowing the use of a longer needle. The formula for the galvanometer (the proof for which will be found in text-books on physics) is

$$i = H\frac{r}{2\pi n}\left(1+\frac{x^2}{r^2}\right)^{\frac{3}{2}}\tan\Theta.$$

(If a simpler type of tangent galvanometer, with but one coil, is used, x is the distance from the plane of this coil to the suspension of the needle.)

Equating this expression, which involves the dimensions of the galvanometer and the deflection, to the current as determined by the voltameter, the horizontal component is deduced.

When the adjustments have been completed, open the switch, remove the trial cathode and put in place the other cathode, which has been kept wrapped in paper. Take care that there is no metallic connection between the cathode and the anode plates. *Remove all iron from the neighborhood of the tangent galvanometer and from your pockets.* All wires must be close together to avoid stray induction and the galvanometer had best be at some distance from the other apparatus.

After reading the temperature of the calorimeter every minute for five minutes note the exact second on an ordinary watch, and close the switch. As soon as possible, read both ends of the needle. Reverse the current, *making the reversal quickly*, and again read both ends of the needle. Always

estimate tenths. Reading both ends of the needle eliminates error due to the axis about which the pointer turns, not coinciding with the center of the graduated circle, and reversing eliminates uncertainty about the reading for the zero position. Keep the current constant with the variable resistance. At intervals of three minutes (approximately) read both ends of the needle and, reversing, again read both ends. Read the temperature every minute to tenths of the smallest division. Allow the current to flow until the temperature has risen to the extent desired. Note the exact second of breaking the circuit. Continue to observe the temperature at minute intervals for five minutes. Remove the cathode, being very careful not to touch the copper deposit. Wash it gently with tap water and then with alcohol, allowing the liquid to simply flow over the surface. When it is dry, weigh as before. Measure very carefully the diameter of the coils in a number of directions, and from the mean determine r. Count n, the total number of turns in both coils, and measure $2x$, the distance between the centers of the two coils. Weigh the inner calorimeter vessel and note of what metal it consists. Plot the temperature readings and correct for radiation (p. 63).

Questions.

1. Calculate what the exact voltage at the terminals of the heating coil was.
2. What sources of error remain uneliminated?
3. Calculate the mean activity of the current in the coil during the experiment.
4. What are the peculiarities and advantages of the tangent galvanometer?
5. What chemical actions take place in the voltameter? To what is the deposition of any metal proportional?
6. Why is it advantageous to have the deflection about 45°?

LXVIII. THERMOELECTRIC CURRENTS.

Text-book of Physics (Duff), pp. 593–598; *Ames' General Physics*, pp. 679–683; *Watson's Physics*, pp. 696–705; *Hadley's Electricity and Magnetism*, pp. 359–381.

To the ends of wires of iron, nickel, silver etc., copper wires are soldered and brought to binding posts on a

board. Below the ends of the board are vessels containing sand or oil in which two test-tubes are supported. The junctions are placed in these test-tubes as indicated in figure 75. The binding posts are connected, by copper wires, to a key of as many parts as there are wires to be tested, so that each circuit may be completed through a sensitive galvanometer. Thermometers are placed in the test-tubes to note the temperatures as one vessel is being heated by a burner.

It is especially important that the temperature should be ascertained accurately. Hence heat should be applied

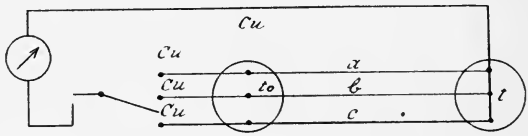

FIG. 75.

cautiously, especially at first, and, when observations are to be made, the source of heat should be removed, and time should be allowed for the temperatures to become fairly constant.

The galvanometer reading should be noted with the greatest care and the zero should be frequently tested. After each reading of the galvanometer, the temperature should be noted.

If a high resistance galvanometer of sufficient sensitiveness is available, the other resistances may be neglected and the various e. m. f.'s will then be proportional to the deflections. Or a sensitive low resistance galvanometer, with a constant high resistance permanently in series, may be used with similar simplicity. The constant of the galvanometer, considered as a voltmeter, may be found by applying to it a fraction of the e. m. f. of a standard cell (pp. 161, 179).

With this arrangement (which will be readily understood from the figure) the thermo-electric force of each circuit, consisting of copper and another wire, may be de-

termined. Curves representing the results should be plotted with the differences of temperature of the junctions as abscissæ and the e. m. f's as ordinates.

If a low resistance galvanometer of low sensitiveness is used, it will be necessary to consider it as an ammeter. In this case, the resistances of the various circuits and of the galvanometer must be found and the constant of the galvanometer, in amperes per unit deflection, must be obtained by connecting it in series with a standard cell and a sufficient known resistance. Thus, the currents and the resistances being known, the thermo-electric forces can be calculated.

Questions.

1. State what would be observed if the temperature of the hot junction were increased steadily beyond the highest temperature used in this experiment.
2. Is the effect observed here due solely to differences of potential produced at the contacts?

LXIX. ELEMENTARY STUDY OF RESISTANCE, SELF-INDUCTION AND CAPACITY.

Text-book of Physics (*Duff*), pp. 623–624; *Watson's Physics*, p. 758; *Hadley's Electricity and Magnetism*, pp. 441–450.

In the following exercises, which are intended for students who have not made a study of the theory of alternating currents, some of the properties of such currents are studied and compared with those of direct currents.

Ohm's Law for steady currents states that

$$\frac{E}{i} = \text{a constant, } R,$$

where R is called the resistance of the conductor.

(A) Apply various e. m. f.'s to a non-inductive conductor. Measure the current by an ammeter, and the voltage by a voltmeter (of any type) and calculate R for each value of the e. m. f. The latter may be varied by means of a series rheostat.

(B) Apply the same method to (1) a large coil, (2) the large coil and the non-inductive resistance, (a) in series, and, (b) in parallel. Compare the results of (a) and (b) with the calculated values.

(C) Repeat (A) and (B), using alternating currents and an electrostatic voltmeter. Corresponding to Ohm's Law we have

$$\frac{E}{i} = \text{constant} = \sqrt{R^2 + 4\pi^2 n^2 L^2},$$

where the constant is called the impedance and L is the coefficient of self-induction and n the frequency. Find the value of the constant for different e. m. f.'s and currents, and, from the mean and the values of R and n, calculate L. Contrast the results in series and parallel combinations with the values calculated by treating impedance in the same way as resistance in direct currents.

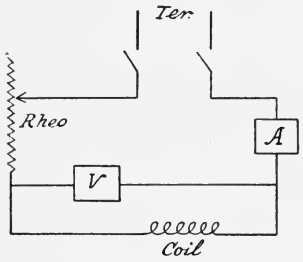

FIG. 76.

Tabulate all results so that they may be readily compared.

(D) When an alternating current is applied to a condenser, it is charged, discharged, charged oppositely and discharged during each alternation. Evidently the total quantity that traverses the leads in each unit of time is proportional to the frequency and to the product of the capacity and the voltage (since $q = CV$) and it can be shown that the current is given by

$$i = 2\pi n C V.$$

Measure i for various values of V and calculate C. Do this for several condensers (1) separately, (2) in parallel, (3) in series. Compare the results of (2) and (3) with the calculated values.

Questions.

1. What is meant by the effective value of an alternating current and what ratio does it bear to the maximum value?

2. How, by means of a diagram, would you find the impedance when given the ohmic resistance R and the inductance L?

3. Supposing the alternating e. m. f. resolved graphically into two parts, one to overcome the ohmic resistance and the other to overcome the inductance, what relation between the phases of these two parts does question (2) suggest?

LXX. SELF-INDUCTION, MUTUAL INDUCTION AND CAPACITY, ALTERNATING CURRENTS.

See references to LXIX. J. J. Thomson's Electricity and Magnetism, §§233, 244–245; *Jackson's Alt. Cur.*, pp. 90–91, 151–200; *Parr's Electrical Eng. Testing*, pp. 222–224, 228–231, 234–235. *For Electrostatic Voltmeter, see Parr*, 367–371.

This exercise, which is somewhat more advanced than the preceding, is intended for students who have made some study of the theory of alternating currents.

Let E be the alternating e. m. f. in a circuit of resistance R, capacity C, and self-induction L. If i is the current,

$$i = \frac{E}{\sqrt{R^2 + (L\omega - 1/C\omega)^2}},$$

We can test the above formula by calculation, after measuring i, E, R, C, and L. An inductance with a magnetic core has a variable value of L, the magnitude of which depends on the strength of the current. Hence, for this experiment, an inductance consisting of a very large coil containing no iron is used.

(A) *Measurement of C.*—If, in the general formula, L be zero, C can be deduced from the values of i, E, and R, assuming that ω, which equals 2π times the frequency n, is known. E, the e. m. f. across the terminals of the condenser, is measured by an electrostatic voltmeter, i by an alternating current ammeter. Initially a high resistance of large current capacity must be included. This may later be cut out. A fuse of lower capacity than the range of the ammeter must be permanently in circuit.

(B) *Measurement of L.*—The value of L is found by observing the values of i and E in a circuit containing the self-inductance coil and then applying the general formula.

Sufficient additional resistance must be placed in the circuit, but the value of E required is that across the terminals of the inductance coil. R, which in this case is the resistance of the coil, is best found by Wheatstone's Bridge.

(C) *Test of General Formula.*—Connect the condenser and self-induction *in series.* Measure the current and the total e. m. f.; also the e. m. f. across each part. Connect the condenser and inductance coil *in parallel.* Measure the common e. m. f., the total current and the current in each branch.

Calculate i in the series arrangement from the above formula and compare with the experimental value. If you are familiar with the method of complex quantities and graphical methods, apply these also to calculate the currents in both series and parallel arrangements.

(D) *Measurement of Mutual Inductance.*—Measure the mutual inductance, M, of the two coils of a transformer (with iron core) by observing the e. m. f., E, across one coil when a measured current, i, is applied to the other.

$$E = Mi\omega.$$

Vary i several times and find how M varies.

Questions.

1. Why is an electrostatic voltmeter necessary?
2. Does the self-induction depend upon the frequency? Why does the latter enter into the equation?

LXXI. DIELECTRIC CONSTANTS OF LIQUIDS.

Text-book of Physics (*Duff*), pp. 533, 535; *Ames' General Physics*, pp. 641, 661; *Watson's Physics*, p. 637; *Hadley's Electricity and Magnetism*, Chapter X.

The dielectric constant of a liquid, or the ratio of the capacity of a condenser with that liquid as dielectric to its capacity when its dielectric is air, can be determined by a comparison of capacities by the Bridge Method of Exp. LX. For this purpose it is convenient to use a condenser consisting of two parallel plates, the distance between which

is adjustable, as shown in figure 77. The distance between the plates can be measured by means of a scale, B, attached to the movable plate A, and a vernier attached to the framework. The plates hang in a vessel for holding the dielectric. Two methods can be used. In one the distance between the plates is not varied; in the other it is varied.

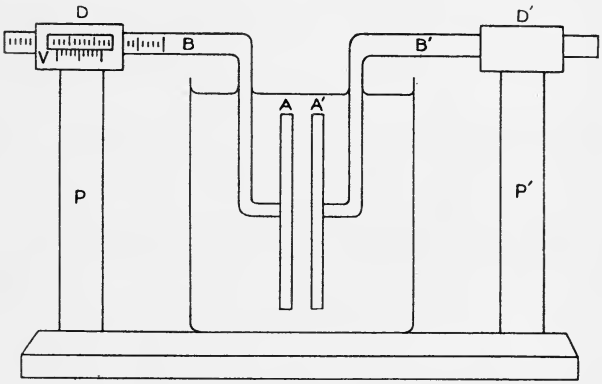

FIG. 77.

The *first method* consists in comparing the capacity of the above condenser with that of a Leyden jar (1) when the dielectric is the liquid to be tested; (2) when it is air. From these comparisons the ratio of the capacities of the condenser in the two conditions is deduced and this equals the dielectric constant of the liquid. Instead of a battery and galvanometer, an induction coil and a sensitive telephone are used.

The *second method* assumes the (approximate) formula

$$C = \frac{Ae}{4\pi d}$$

for the capacity of such a plate condenser (in electrostatic units, see p. 162), where A is the area of each plate, e is the dielectric constant of the surrounding medium, and d is

their distance apart. If C be the same with two dielectrics, but with different values of d

$$\frac{e_1}{e_2} = \frac{d_1}{d_2}$$

Having obtained a balance for air as dielectric, leave the resistances in the bridge unchanged and again obtain a balance, after filling the jar with the liquid to be tested, by adjusting the distance between the plates. This second method is less accurate, since the formula assumed is only approximate and the distances cannot be determined as accurately as the resistances. An accurate formula will be found in Kohlrausch's Physical Measurements, p. 379.

The above methods should be applied to two highly insulating liquids, such as kerosene and benzol.

Questions.

1. Calculate the capacity of the two plates when separated by (*a*) air; (*b*) liquid, the distance apart being the same as in the first part of the experiment.
2. Calculate the charge for each case if (*a*) 100 electrostatic units of potential are applied; (*b*) 100 volts (p. 162).

LXIII. ELECTRIC WAVES ON WIRES.

Dielectric Constants of Liquids.

Ames' General Physics, pp. 752–754; *Watson's Physics*, pp. 856–858, 870–871; *Text-book of Physics (Duff)*, pp. 635–639; *Hadley's Magnetism and Electricity*, pp. 541–548, 585–586; *J. J. Thomson's Electricity and Magnetism*, § 243; *Kohlrausch's Physical Measurements* (on capacity of a plate condenser), p. 379. *Drude, An. der Phys.*, Vol. 8, p. 336.

In this experiment electric waves on a wire, AD, are excited by electric oscillations in a neighboring circuit or "exciter," E, which contains an inductance, L, and a capacity, C. The period of such oscillations is $T = 2\pi\sqrt{LC}$. The inductance is that of two thick semicircular wires. The ends e and e' of these wires carry small spheres and the wires are so bent that the spheres are beneath the

surface of kerosene in a small cup and form a spark-gap. The length of this spark-gap can be adjusted by means of a micrometer screw attached to one of the ebonite posts, *H H′*, on which the semicircular wires are supported. The condenser is of the variable form shown in figure 77 and is connected between the other two ends of the semicircular wires. The impulses that start the oscillations in the exciter are produced by a Tesla coil, the secondary of which is connected across the spark-gap *ee′*, while the primary of the Tesla coil is connected through another spark-gap, *Z*, to the secondary of an induction coil *I*.

The wire, *AD*, on which the waves are formed is bent to a *U*-form and lies in a horizontal plane above the plane of the exciter. The oscillations in the exciter act by induction on the part, *A*, of the wire and produce waves that move along the wire toward *D;* between any two corresponding points, such as *d* and *d′*, there is an oscillating difference of potential and the transmission of this oscillation constitutes the wave-motion. At the free end, *D*, these waves are reflected and interfere with the direct waves. If the wire is of proper length, this interference

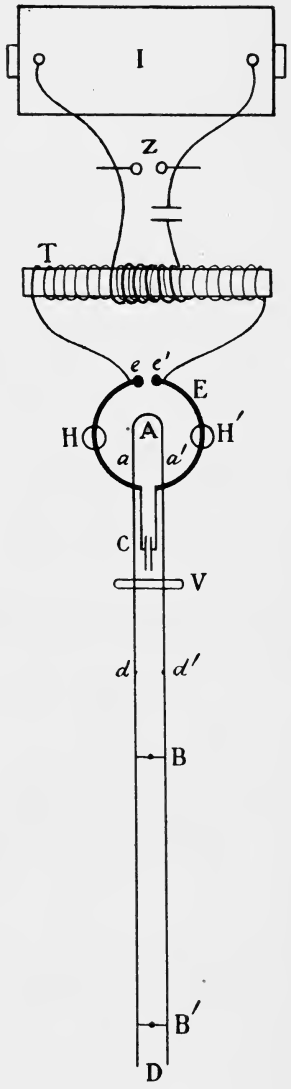

FIG. 78.

produces stationary waves; that is, the wire "resonates" to the exciter. If the wire be also sufficiently long, there will be one or more nodes on the wire, that is, places of no potential difference, with intervening antinodes, or places of maximum oscillating potential difference. If a small wire "bridge," B, be placed across the wire at a node it will not interfere with the stationary waves; but it will destroy them if it is placed at any other point. (It will be instructive to compare the above with the formation of stationary sound waves in a resonance tube such as that of Exp. XXX, the tuning-fork being the exciter.) When the bridge has been placed at a node the part of the wire between it and the free end, *D*, could be changed in length or removed without appreciably diminishing the oscillations between *A* and *B*, (just as the lower part of a violin string that is touched by the finger does not interfere with the vibrations of the upper part). The part *A* of the wire is (approximately) a node, although it is the part where the oscillations are excited. (Compare with this the fact that when a tuning-fork, connected to a long thread as in Melde's experiment, throws the latter into stationary vibrations, the point of connection is a node).

Various means have been used for detecting such stationary vibrations. The simplest is a "vacuum" tube, *V*, which contains helium at a very low pressure. An alternating potential difference between the ends of such a tube will cause oscillating discharges accompanied by a glow. If placed across the wire when there are no stationary oscillations the tube will not glow; but, if stationary oscillations exist, it will glow brightly at an antinode, less brightly between an antinode and a node, not at all at a node. By the aid of the tube the bridge can be adjusted to each node. If the wire be long enough to permit of more than one node, twice the distance between two adjacent nodes will equal the wave length.

Since the wire is in resonance with the exciter, the period of oscillation of the wire equals that of the exciter. The

latter, and therefore the former, can be changed by changing C. Hence

$$\frac{C_1}{C_2}=\frac{T_1^2}{T_2^2}.$$

The electric waves, while directed by the wire, are really waves of oscillation of electric force in the medium between the two branches of the wire. Such waves travel with a velocity that is independent of the wave length, and, if λ_1 be the wave-length when the period is T_1, λ_2 that when the period is T_2, $v=\lambda_1/T_1=\lambda_2/T_2$. Hence

$$\frac{C_1}{C_2}=\frac{\lambda_1^2}{\lambda_2^2}$$

By determining the wave-length with air as the dielectric in C and then with a liquid as dielectric, we can evidently find the dielectric constant of the liquid.

In the practice of the method most trouble is likely to be due to the spark-gap, *ee'*. It must be adjusted until the spark occurs under the kerosene. To avoid danger to the tube by accidental dropping, it may be attached loosely to the wire by a loop of thread. Each node should be determined several times. The distance between the plates of C should be varied four or five times with air as dielectric and a curve drawn with d as abscissa and λ as ordinate. From this curve and the value of λ for each liquid, the value of the dielectric constant for that liquid is readily deduced.

For a complete proof of the formula works on electricity and magnetism must be consulted (e. g., J. J. Thomson's Electricity and Magnetism, § 243). The following considerations *suggest* the formula. When an alternating e. m. f., E, is applied to a circuit containing capacity, self-inductance, and ohmic resistance in series,

$$E=I\sqrt{r^2+(\omega L-\tfrac{1}{\omega C})^2}$$

The potentials across the self-inductance (including the resistance) and the capacity, respectively, are

$$E_L=I\sqrt{r^2+\omega^2L^2} \text{ and } E_C=\frac{I}{\omega C}$$

Evidently the potentials across the parts of the system may be greater than the total e. m. f. which is the resultant obtained by geometrical (i. e., vector) addition of the parts. This constitutes resonance. It is complete when

$$\omega L - \frac{1}{\omega C} = 0$$

Substituting for ω its value $2\pi/T$, we get $T = 2\pi\sqrt{LC}$. This, then, is the period of the applied e. m. f. when resonance results. It is also, therefore, the period of the free natural vibrations of the system.

The approximate formula for the capacity of a plate condenser is given in Exp. LXII, a more exact one in Kohlrausch, p. 379.

Questions.

1. Assuming the velocity of the waves to be that of light, calculate the frequency of the oscillations for one value of λ.
2. From this and the approximate formula for C calculate L.
3. If a sufficient amount of liquid were available, how could its dielectric constant be found by immersing the wire AD in it?
4. What would be observed if AD were contained in a vacuum tube?

TABLES.

TABLE I.

Logarithms of Numbers from 1 to 1000.

No.	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5515	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
No.	0	1	2	3	4	5	6	7	8	9

TABLE I.—Continued.

Logarithms of Numbers from 1 to 1000.

No.	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
No.	0	1	2	3	4	5	6	7	8	9

TABLE II.

Natural Sines and Cosines.

	Sine	D 1°		Cosine	D 1°
↓ 0	0.0000		90	1.0000	
1	0.0175	175	89	0.9998	02
2	0.0349	174	88	0.9994	04
3	0.0523	174	87	0.9986	08
4	0.0698	175	86	0.9976	10
5	0.0872	174	85	0.9962	14
6	0.1045	173	84	0.9945	17
7	0.1219	174	83	0.9925	20
8	0.1392	173	82	0.9903	22
9	0.1564	172	81	0.9877	26
10	0.1736	172	80	0.9848	29
11	0.1908	172	79	0.9816	32
12	0.2079	171	78	0.9781	35
13	0.2250	171	77	0.9744	37
14	0.2419	169	76	0.9703	41
15	0.2588	169	75	0.9659	44
16	0.2756	168	74	0.9613	46
17	0.2924	168	73	0.9563	50
18	0.3090	166	72	0.9511	52
19	0.3256	166	71	0.9455	56
20	0.3420	164	70	0.9397	58
21	0.3584	164	69	0.9336	61
22	0.3746	162	68	0.9272	64
23	0.3907	161	67	0.9205	67
24	0.4067	160	66	0.9135	70
25	0.4226	159	65	0.9063	72
26	0.4384	158	64	0.8988	75
27	0.4540	156	63	0.8910	78
28	0.4695	155	62	0.8829	81
29	0.4848	153	61	0.8746	83
30	0.5000	152	60	0.8660	86
31	0.5150	150	59	0.8572	88
32	0.5299	149	58	0.8480	92
33	0.5446	147	57	0.8387	93
34	0.5592	146	56	0.8290	97
35	0.5736	144	55	0.8192	98
36	0.5878	142	54	0.8090	102
37	0.6018	140	53	0.7986	104
38	0.6157	139	52	0.7880	106
39	0.6293	136	51	0.7771	109
40	0.6428	135	50	0.7660	111
41	9.6561	133	49	0.7547	113
42	0.6691	130	48	0.7431	116
43	0.6820	129	47	0.7314	117
44	0.6947	127	46	0.7193	121
45	0.7071	124	45° ↑	0.7071	122
	Cosine	D 1°		Sine	D 1°

TABLE III.

For Reduction of Time of Oscillation to an Infinitely Small Arc.

$$k = \frac{1}{4}\sin^2\frac{a}{4} + \frac{5}{64}\sin^4\frac{a}{4}.$$

If t = observed time and
T = true reduced time
$T = t - kt$.

a	k	a	k	a	k
0°	0.00000	7°	0.00023	14°	0.00093
1	000	8	030	15	107
2	002	9	039	16	122
3	004	10	048	17	138
4	008	11	058	18	154
5	012	12	069	19	172
6	017	13	080	20°	190
7°	023	14°	093		

TABLE IV.

Reduction of Barometer Readings to o.

(The corrections below are in mm. and are to be subtracted. The uncorrected height is in cm.)

Temp.	Brass Scale							Glass Scale				
	72	73	74	75	76	77	78	74	75	76	77	78
15	1.75	1.77	1.81	1.83	1.86	1.88	1.91	1.92	1.94	1.97	2.00	2.02
16	1.87	1.89	1.93	1.96	1.98	2.01	2.03	2.05	2.07	2.10	2.13	2.16
17	1.98	2.01	2.05	2.08	2.10	2.13	2.16	2.17	2.20	2.23	2.26	2.29
18	2.10	2.13	2.17	2.20	2.23	2.26	2.29	2.30	2.33	2.36	2.39	2.43
19	2.22	2.25	2.29	2.32	2.35	2.38	2.41	2.43	2.46	2.49	2.53	2.56
20	2.33	2.37	2.41	2.44	2.47	2.51	2.54	2.56	2.59	2.62	2.66	2.69
21	2.45	2.48	2.53	2.56	2.60	2.63	2.67	2.68	2.72	2.76	2.79	2.83
22	2.57	2.60	2.65	2.69	2.72	2.76	2.79	2.81	2.85	2.89	2.92	2.96
23	2.68	2.72	2.77	2.81	2.84	2.88	2.92	2.94	2.98	3.02	3.06	3.10
24	2.80	2.84	2.89	2.93	2.97	3.01	3.05	3.06	3.11	3.15	3.19	3.23
25	2.92	2.96	3.01	3.05	3.09	3.13	3.17	3.19	3.23	3.28	3.32	3.36

TABLE V.

Density and Volume of One Gram of Water at Different Temperatures.

Temp.	Density	Vol. of 1. gr.	Temp.	Density	Vol. of 1. gr.
0°	0.999878	1.000122	21°	0.998065	1.001939
1	0.999933	1.000067	22	0.997849	1.002156
2	0.999972	1.000028	23	0.997623	1.002383
3	0.999993	1.000007	24	0.997386	1.002621
4	1.000000	1.000000	25	0.997140	1.002868
5	0.999992	1.000008	30	0.99577	1.00425
6	0.999969	1.000031	35	0.99417	1.00586
7	0.999933	1.000067	40	0.99236	1.00770
8	0.999882	1.000118	45	0.99035	1.00974
9	0.999819	1.000181	50	0.98817	1.01197
10	0.999739	1.000261	55	0.98584	1.01436
11	0 999650	1.000350	60	0.98334	1.01694
12	0.999544	1.000456	65	0.98071	1.01967
13	0.999430	1.000570	70	0.97789	1.02261
14	0.999297	1.000703	75	0.97493	1.02570
15	0.999154	1.000847	80	0.97190	1.02891
16	0.999004	1.000997	85	0.96876	1.03225
17	0.998839	1.001162	90	0.96549	1.03574
18	0.998663	1.001339	95	0.96208	1.03941
19	0.998475	1.001527	100	0.95856	1.04323
20	0.998272	1.001731			

TABLE VI.

Density of Gases (0°, 76 cm.).[1]

Hydrogen	.00008987
Oxygen	.0014290
Nitrogen	.0012507
Air	.0012928
Chlorine	.003167
Carbon monoxide	.0012504
Carbon dioxide	.0019768
Ethane	.001341
Ethylene	.001252
Steam (at 100°)	.00060315

[1] Largely from Guye, J. Ch. Phys., 1907, p. 203.

TABLE VII.

Density (0°), Specific Heat (0°), and Coefficient of Linear Expansion.

Element	Density	Specific Heat	Coef. of Lin. Exp. Multiplied by 10^6
Aluminum	2.60	.22	23.1
Antimony	6.62	.049	.....
Bismuth	9.8	.031	.....
Cadmium	8.61	.055	30.7
Carbon, diamond	3.52	.10	1.18
Carbon, graphite	2.25	.15	7.8
Carbon, gas carbon	1.90	.....	5.4
Cobalt	8.8	.106	12.4
Copper	8.92	.094	16.8
Copper sulphate (crys.)	3.58	.....	.....
Gold	19.3	.032	14.4
Iron	7.8	.11	12.1
Lead	11.36	.029	29.2
Magnesium	1.74	.....	.....
Mercury	13.596	.0333	181 (cub. exp.)
Nickel	8.9	.108	12.8
Phosphorus, yellow	1.83	.20	.....
Phosphorus, red	2.19	.17	.....
Phosphorus, metallic	2.34	.....	.....
Platinum	21.4	.033	9.0
Potassium chloride	1.98	.....	.....
Silver	10.53	.056	19.2
Sodium chloride	2.15	.....	.....
Sodium sulphate	2.65	.....	.....
Tin	7.3	.056	22.3
Zinc	7.2	.094	29.2
Zinc sulphate (anhy.)	3.49	.....	.....

TABLE VIII.

Density, Specific Heat, and Coefficient of Expansion of Miscellaneous Substances (0°).

Substance	Density	Specific heat	Coef. of Lin. Exp. ($\times 10^6$)
Castor oil	.969		
Glass, green	2.6	.19	8.9
Glass, crown	2.7	.19	8.8
Glass, crystal	2.9	.18	7.7
Glass, flint	3.15–3.9	.19	7.3
Hard rubber	1.15		7.7
Marble	2.75		11.7
Paraffin	.89		
Quartz, crystal II	2.653	.19	7.2
Quartz, crystal ⊥	2.653		13.2
Quartz, fused	2.20		.54
Alcohol (ethyl)	.81	.54	1.048[1]
Benzol	.899	.38	1.176[1]
Carbon bisulphide	1.293	.24	1.14[1]
Chloroform	1.53	.23	1.11[1]
Ether (ethyl)	.74	.53	1.51[1]
Glycerine	1.26	.58	

TABLE IX.

Average Value of Elastic Moduli.

1	*Shear Modulus.*	*Young's Modulus.*
Brass	3.7×10^{11}	10.4×10^{11}
Iron	7.7×10^{11}	19.6×10^{11}
Steel	8.2×10^{11}	22×10^{11}

[1] Coefficient of *cubical* expansion $\times 10^3$.

TABLE X.

Surface Tension T (15°), Temperature Coefficient of Surface Tension c′, and Angle of Contact α.

	T	c′	α
Ethyl ether	19	—.11	16°
Ethyl alcohol	25	—.087	0°
Benzol	31	—.13	0°
Water	76	—.15	small
Mercury	527	—.38	135°

TABLE XI.

Coefficient of Viscosity (20°).[1]

Water	.0100
Mercury	.0159
Acetic acid	.0122
Methyl alcohol	.00591
Ethyl alcohol	.0119
Ethyl ether	.00234
Benzol	.00649

[1] Winkelmann, 1908, I, 2, p. 1397.

TABLE XII.

Specific Heats of Gases.[1]

	Temp.	s_p	$\gamma = \frac{s_p}{s_v}$
Argon	20°	.1205	1.66
Helium	20°	1.25	1.64
Mercury	275°–356°	.0246	1.66
Hydrogen	0°–200°	3.406	1.396
Nitrogen	−30°–200°	.244	1.405
Oxygen	0°–200°	.217	1.40
Air	0°–200°	.2375	1.405
Chlorine	19°–343°	.115	1.32
Iodine	200°–377°	.0336	1.29
Bromine	85°–228°	.0555	1.29
Water	130°–250°	.480	1.287
Hydrogen sulphide	10°–200°	.245	1.28
Carbon dioxide	100°	.217	1.28
Ammonia	20°–210°	.512	1.317
Chloroform	28°–118°	.144	1.154
Ethyl alcohol	110°–220°	.453	1.14
Ether	70°–225°	.480	1.07
Benzol	116°–218°	.375	1.187

[1] Jüptner, Phys. Chem. I, pp. 71–73.

TABLE XIII.

Pressure of Saturated Water Vapor (Regnault).
(mm.)

Temp.	Pressure		Temp.	Pressure
	Ice	Water	29°	29.782
			30	31.548
−10	1.999	2.078	31	33.405
			32	35.359
8	2.379	2.456	33	37.410
			34	39.565
6	2.821	2.890	35	41.827
			40	54.906
4	3.334	3.387	45	71.391
			50	91.982
2	3.925	3.955	55	117.479
			60	148.791
			65	186.945
0		4.600	70	233.093
+ 1		4.940	75	288.517
2		5.302	80	354.643
3		5.687	85	433.41
4		6.097	90	525.45
5		6.534	91	545.78
6		6.998	92	566.76
7		7.492	93	588.41
8		8.017	94	610.74
9		8.574	95	633.78
10		9.165	96	657.54
11		9.792	97	682.03
12		10.457	98	707.26
13		11.062	98.5	720.15
14		11.906	99.0	733.91
15		12.699	99.5	746.50
16		13.635	100.0	760.00
17		14.421	100.5	773.71
18		15.357	101.0	787.63
19		16.346	102.0	816.17
20		17.391	104.0	875.69
21		18.495	105	906.41
22		19.659	110	1075.4
23		20.888	120	1491.3
24		22.184	130	2030.3
25		23.550	150	3581.2
26		24.998	175	6717
27		26.505	200	11690
28		28.101	225	19097

TABLE XIV.

Boiling Point of Water, t, at Barometric Pressure, p, (m m.).

p.	*t.*	*p.*	*t.*	*p.*	*t.*
740	99.26°	750	99.63°	760	100.00°
41	.29	51	.67	61	.04
42	.33	52	.70	62	.07
43	.37	53	.74	63	.11
44	.41	54	.78	64	.15
45	.44	55	.82	65	.18
46	.48	56	.85	66	.22
47	.52	57	.89	67	.26
48	.56	58	.93	68	.29
49	.59	59	.96	69	.33
750	99.63°	760	100.00°	770	100.36°

TABLE XV.

Wet and Dry Bulb Hygrometer.

(Actual vapor pressures (mm.) for different temperatures of dry thermometer and various differences of temperature between the two thermometers.

The first vertical column gives the temperature of the dry-bulb thermometer. The first horizontal line gives the difference between the two thermometers. Since the difference is zero if the air is saturated, the second vertical column gives the saturated vapor pressure for the corresponding temperatures in the first column.)

t°C.	0	1	2	3	4	5	6	7	8	9	10	11
0	4.6	3.7	2.9	2.1	1.3							
1	4.9	4.0	3.2	2.4	1.6	0.8						
2	5.3	4.4	3.4	2.7	1.9	1.0						
3	5.7	4.7	3.7	2.8	2.2	1.3						
4	6.1	5.1	4.1	3.2	2.4	1.6	0.8					
5	6.5	5.5	4.5	3.5	2.6	1.8	1.0					
6	7.0	5.9	4.9	3.9	2.9	2.0	1.1					
7	7.5	6.4	5.3	4.3	3.3	2.3	1.4	0.4				
8	8.0	6.9	5.8	4.7	3.7	2.7	1.7	0.8				
9	8.6	7.4	6.3	5.2	4.1	3.1	2.1	1.1	0.2			
10	9.2	8.0	6.8	5.7	4.6	3.5	2.5	1.5	0.5			
11	9.8	8.6	7.4	6.2	5.1	4.0	2.9	1.9	0.9			
12	10.5	9.2	8.0	6.8	5.6	4.5	3.4	2.3	1.3			
13	11.2	9.8	8.6	7.3	6.2	5.0	3.9	2.8	1.7			
14	11.9	10.6	9.2	8.0	6.7	5.6	4.4	3.3	2.2	1.1		
15	12.7	11.3	9.9	8.6	7.4	6.1	5.0	3.8	2.7	1.6	0.5	
16	13.5	12.1	10.7	9.3	8.0	6.8	5.5	4.3	3.2	2.1	1.0	
17	14.4	13.0	11.5	10.1	8.7	7.4	6.2	4.9	3.7	2.6	1.5	0.4
18	15.4	13.8	12.3	10.9	9.5	8.1	6.8	5.5	4.3	3.1	2.0	0.9
19	16.4	14.7	13.2	11.7	10.3	8.9	7.5	6.2	4.9	3.7	2.5	1.4
20	17.4	15.7	14.1	12.6	11.1	9.7	8.3	6.9	5.6	4.3	3.1	1.9
21	18.5	16.8	15.1	13.5	12.0	10.5	9.0	7.6	6.3	5.0	3.7	2.5
22	19.7	17.9	16.2	14.5	12.9	11.4	9.9	8.4	7.0	5.7	4.4	3.1
23	20.9	19.0	17.3	15.6	13.9	12.3	10.8	9.2	7.8	6.4	5.1	3.8
24	22.2	20.3	18.4	16.6	14.9	13.3	11.7	10.1	8.7	7.2	5.8	4.5
25	23.6	21.6	19.7	17.8	16.0	14.3	12.7	11.1	9.5	8.0	6.6	5.2
26	25.0	22.9	21.0	19.0	17.2	15.4	13.7	12.1	10.5	8.9	7.4	6.0
27	26.5	24.9	22.3	20.3	18.4	16.6	14.8	13.1	11.4	9.8	8.3	6.8
28	28.1	25.9	23.7	21.7	19.7	17.6	16.0	14.2	12.5	10.8	9.2	7.7
29	29.8	27.5	25.3	23.1	21.1	19.1	17.2	15.3	13.6	11.9	10.2	8.6
30	31.6	29.2	26.9	24.6	22.5	20.5	18.5	16.6	14.7	13.0	11.2	9.6

TABLE XVI.

Vapor Pressure of Mercury (mm.).

Temp.	Pres.	Temp.	Pres.
0	0.02	170	8.091
+20	0.04	180	11.000
40	0.08	190	14.84
60	0.16	200	19.90
80	0.35	210	26.35
100	0.746	220	34.70
110	1.073	230	45.35
120	1.534	240	58.82
130	2.175	250	75.75
140	3.059	260	96.73
150	4.266	270	123.01
160	5.900	280	155.17

TABLE XVII.

Melting Point of Metals. (Holborn and Day and Waidner and Burgess.[1])

Tin	232
Cadmium	321
Lead	327
Zinc	419
Antimony	631
Aluminum	657
Silver	961
Gold	1063
Copper	1084
Platinum	1770

[1] Phys. Rev., 1909, p. 467; Compt. Rend., 1909, cxlviii, p. 1177.

TABLE XVIII.

Wave Lengths in Angstrom Units (10^{-8} cm.).

Line	Element	Wave Length	Color
C, H_α	Hydrogen	6563.054	Red
D_1	Sodium	5896.155	Yellow
D_2	Sodium	5890.182	Yellow
F, H_β	Hydrogen	4861.527	Blue
G′ H_γ	Hydrogen	4340.634	Violet
H	Calcium	3968.625	Violet
	Helium	7065.2	Red
	Helium	6678.1	Red
	Helium	5875.6	Yellow
	Helium	5015.7	Green
	Helium	4921.9	Blue
	Helium	4713.2	Blue
	Helium	4471.5	Violet
	Mercury	6232.0	Red
	Mercury	5790.7	Yellow
	Mercury	5769.6	Yellow
	Mercury	5460.7	Green
	Mercury	4959.7	Green–Blue
	Mercury	4916.4	Blue
	Mercury	4358.3	Blue
	Mercury	4078.1	Violet
	Mercury	4046.8	Violet
K_α	Potassium	7699.3	Red
K_β	Potassium	5832.2	Yellow
K_γ	Potassium	4047.4	Violet
Li_α	Lithium	6708.2	Red
Li_β	Lithium	6103.8	Orange
	Cadmium	6438.5	Red
	Cadmium	5085.8	Green
	Cadmium	4799.9	Blue

TABLE XIX.

Refractive Indices.

[Yellow light, (D lines) 20°.]

Glass, light crown (density = 2.50)	1.5280
Glass, heavy crown (density = 3.00)	1.5604
Glass, light flint (density = 2.87)	1.5410
Glass, heavy flint (density = 4.22)	1.7102
Quartz, crystal, ⊥, ord	1.5442
Quartz, crystal, ⊥, ext	1.5533
Alcohol, ethyl	1.3614
Benzol,	1.5014
Carbon bisulphide	1.6277
Chloroform	1.4490
Ether, ethyl	1.3560
Glycerine	1.4729
Water	1.3329
Air, 0°, 76 cm	1.000293

TABLE XX.

Specific Rotatory Power (20°). Yellow Sodium Light (D).[1]

Active Substance	Concentration (=c) (gr. in 100 c.c.)	$[\phi]_D^{20}$
Cane-sugar, *R*	3–28	66.639 − .0208*c*
	10–86	66.453 − .000124*c*
Invert sugar, *L*	1–14	−20.07 − .041*c*
Glucose (dextrose). *R* (crystallized)	0–100%	47.73 + .015 × %
Fructose (levulose) *L*	0–40	−100.3 + .108*c*
Milk-sugar, *R*	5–7	52.53
Tartaric acid, *R*	.5–15	15.06 − .131*c*
	22–63	13.436 − .119*c*
Quartz, *R* or *L*		21.70 (for 1 mm. thickness)

[1] Landolt and Börnstein.

TABLE XXI.

Photometric Table.

For a 300-part Photometric Bar $\frac{n^2}{(300-n)^2}$.

n	0	1	2	3	4
50	0.0400	0.420	0.0440	0.0460	0.0482
60	.0625	.651	.0678	.0706	.0735
70	.0926	.0961	.0997	.1034	.1072
80	.1322	.1368	.1414	.1643	.1512
90	.1837	.1896	.1957	.2018	.2082
100	.2500	.2576	.2653	.2734	.2815
110	.3352	.3449	.3549	.3652	.3756
120	.4445	.4570	.4698	.4829	.4964
130	.5848	.6009	.6173	.6343	.6516
140	.7656	.7864	.8078	.8296	.8521
150	1.0000	1.027	1.055	1.083	1.113
160	1.306	1.342	1.379	1.416	1.454
170	1.700	1.757	1.806	1.856	1.907
180	2.250	2.313	2.379	2.446	2.516
190	2.983	3.070	3.160	3.253	3.350
200	4.000	4.122	4.285	4.380	4.516
210	5.444	5.621	5.803	5.994	6.192
220	7.563	7.826	8.100	8.387	8.687
230	10.80	11.21	11.64	12.09	12.57
240	16.00	16.68	17.41	18.17	18.98
250	25.00				

n	5	6	7	8	9
50	0.0504	0.0527	0.0550	0.0574	0.0599
60	.0765	.0796	.0827	.0859	.0892
70	.1111	.1151	.1193	.1235	.1278
80	.1563	.1615	.1668	.1723	.1779
90	.2148	.2215	.2283	.2354	.2428
100	.2899	.2985	.3074	.3164	.3256
110	.3864	.3974	.4088	.4204	.4323
120	.5012	.5244	.5389	.5538	.5691
130	.6694	.6877	.7064	.7257	.7454
140	.8752	.8988	.9231	.9481	.9737
150	1.143	1.174	1.205	1.238	1.272
160	1.494	1.535	1.577	1.620	1.664
170	1.960	2.014	2.071	2.129	2.188
180	2.588	2.662	2.739	2.817	2.889
190	3.449	3.552	3.658	3.768	3.882
200	4.656	4.803	4.954	5.111	5.275
210	6.398	6.122	6.835	7.068	7.310
220	9.000	9.327	9.670	10.03	10.40
230	13.07	13.60	14.15	14.74	15.35
240	19.84	20.75	21.72	22.75	23.84
250					

TABLE XXII.

Specific Resistance at o° C. and Temperature Coefficient.

	Specific Resistance	Temperature Coefficient
Bismuth (hard)	132.6×10^{-6}	0.0054
Copper (annealed)	1.590×10^{-6}	0.0043
Copper (hard drawn)	1.622×10^{-6}	.....
German silver ($4Cu+2Ni+1Zn$)	20.24×10^{-6}	0.00027
Iron	10.43×10^{-6}	0.007
Lead (pressed)	19.85×10^{-6}	0.0039
Mercury	94.07×10^{-6}	0.00089
Platinum	8.957×10^{-6}	0.0034
Silver (annealed)	1.521×10^{-6}	0.00377
Silver (hard drawn)	1.652×10^{-6}	.....
Tin	9.565×10^{-6}	0.004

TABLE XXIII.

Specific Resistance and Temperature Coefficient of Solutions (18°).*

	Sp. Res.	Temp. Coef.		Sp. Res.	Temp. Coef.
n HCl	3.32	0.0165	n NaCl	13.45	0.0226
0.1n HCl	28.5		0.1n NaCl	108.1	
0.01n HCl	271.		0.01n NaCl	974.	
n HNO_3	3.23	0.163	n KCl	10.18	0.0217
0.1n HNO_3	28.6		0.1n KCl	89.5	
0.01n HNO_3	272.		0.01n KCl	817.	
n $\frac{1}{2}H_2SO_4$	5.05	0.0164	n $AgNO_3$	14.75	0.0216
0.1n $\frac{1}{2}H_2SO_4$	44.4		0.1n $AgNO_3$	105.7	
0.01n $\frac{1}{2}H_2SO_4$	325.		0.01n $AgNO_3$	922.	
n $C_2H_4O_2$	758.		n $\frac{1}{2}Pb(NO_3)_2$	23.8	
0.1n $C_2H_4O_2$	2170.		0.1n $\frac{1}{2}Pb(NO_3)_2$	129.4	
0.01n $C_2H_4O_2$	6990.		0.01n $\frac{1}{2}Pb(NO_3)_2$	967.	
n NaOH	6.25	0.019	n $\frac{1}{2}ZnSO_4$	37.6	0.025
0.1n NaOH	54.7		0.1n $\frac{1}{2}ZnSO_4$	217.	
0.01n NaOH	500.		0.01n $\frac{1}{2}ZnSO_4$	1362.	
n NH_4OH	1125.		n $\frac{1}{2}CuSO_4$	38.8	0.0225
0.1n NH_4OH	3030.		0.1n $\frac{1}{2}CuSO_4$	223.	
0.01n NH_4OH	10420.		0.01n $\frac{1}{2}CuSO_4$	1385.	

*A normal solution (designated by the subscript n), contains in one liter a number of grams equal to the chemical equivalent (atomic or molecular weight divided by the valency). A solution with the subscript 0.1n has one-tenth this concentration, etc. For exmaple, 0.1n HCl has 3.65 grs. of HCl (gas) in one trile of solution, or that proportion.

TABLE XXIV.

Dielectric Constants.

I		II	
Hydrocyanic acid	96	Ether	4.5
Water	80	Xylol	2.26
Methyl alcohol	33	Benzol	2.2
Ethyl alcohol	25	Toluol	2.2
Ammonia (liquid)	22	Petroleum	2.07
Acetone	17		
Sulphur dioxide	14		
Pyridene	12		

INDEX.

1.6

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